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一类具脉冲的非自治高阶 BAM 神经网络周期解的全局指数稳定性

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摘要: 通过构造 Lyapuonv 函数,利用 M-矩阵理论以及 Yang 不等式技巧,研究了一类含脉冲的非自治高阶 BAM(bi-directional associative memory)神经网络周期解的全局指数稳定性,且推广了相关文献中的结果.

关键词: 高阶 BAM 神经网络;周期解;M-矩阵;脉冲;指数稳定性

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0 引言

双向联想记忆(BAM)神经网络模型自从被 B. Kosko[1] 提出以来,在模式识别、信息的智能处理、最优化问题计算以及复杂控制等工程领域中得到广泛应用,一些低阶 BAM 神经网络的稳定性问题受到了极大关注[2-7]. 相对于低阶神经网络模型,高阶神经网络模型在网络的逼近能力、收敛速度、存储水平和容错能力等方面都具有更强的优势,因而愈来愈引起国内外学者的关注[8-13]. 同时,在实际问题中存在许多脉冲现象,使得脉冲神经网络也日益受到研究者的重视[5-7,14-15]. 文献[11]虽然分析了具脉冲的高阶 BAM 神经网络的稳定性问题,但得到的结果有局限性. 而在实际问题中,由于大量长度大小不等的神经元是并行连接的,神经元之间的联接权也是时变的,因此进一步讨论具脉冲的非自治高阶 BAM 神经网络的动力学性质是有意义的. 本文将研究神经网络模型:

$$
\begin{aligned}
&\mathrm{d}x_i(t)/\mathrm{d}t=-a_i(t)x_i(t)+\sum_{j=1}^{m}c_{ij}(t)f_j(y_j(t-\tau_j))+\\
&\sum_{j=1}^{m}\sum_{l=1}^{m}e_{ijl}(t)f_j(y_j(t-\tau_j))f_l(y_l(t-\tau_l))+I_i(t),\ \Delta x_i(t_k)=\\
&I_k(x_i(t_k))=-\gamma_{ik}x_i(t_k),\ 0<\gamma_{ik}<2,\ k=1,2,\cdots,\\
&\mathrm{d}y_j(t)/\mathrm{d}t=-b_j(t)y_j(t)+\sum_{i=1}^{n}d_{ji}(t)g_i(x_i(t-\sigma_i))+\\
&\sum_{i=1}^{n}\sum_{l=1}^{n}\omega_{jil}(t)g_i(x_i(t-\sigma_i))g_l(x_l(t-\sigma_l))+J_j(t),\\
&\Delta y_j(t_k)=I_k(y_j(t_k))=-\beta_{ik}y_i(t_k),\ 0<\beta_{ij}<2,\\
&k=1,2,\cdots,
\end{aligned}
\tag{1}
$$

其中 $i=1,2,\cdots,n$,$j=1,2,\cdots,m$,$0\leqslant\tau_j\leqslant\tau$,$0\leqslant\sigma_i\leqslant\sigma$ 是传输时滞,$x_i(t)$,$y_j(t)$ 分别表示第 i,j 个神经元的状态量,$a_i(t)>0$,$b_j(t)>0$;$c_{ij}(t)$,$d_{ji}(t)$,$e_{ijl}(t)$,$\omega_{jil}(t)$ 分别表示 1 阶和 2 阶联接权;$f_j(\cdot)$,$g_i(\cdot)$ 为激活函数;$I_i(t)$,$J_j(t)$ 分别是在 t 时刻的外部输入;$\Delta x_i(t_k)=x_i(t_k^+)-x_i(t_k^-)$,$\Delta y_j(t_k)=y_j(t_k^+)-y_j(t_k^-)$ 为 t_k 时刻的脉冲,$t_1<t_2<\cdots$ 是一组严格递增序列且 $\lim\limits_{k\to+\infty}t_k=+\infty$

假设 t_k 处的解为 $(x_1(t_k),\cdots,x_n(t_k),y_1(t_k),\cdots,y_m(t_k))^{\mathrm{T}}=(x_1(t_k-0),\cdots,x_n(t_k-0),y_1(t_k-0),\cdots,y_m(t_k-0))^{\mathrm{T}}$,系统(1)的初始条件为

$$
\begin{cases}
x_i(s)=\varphi_{xi}(s),\ s\in[-\sigma,0],\ i=1,2,\cdots,n,\\
y_j(s)=\varphi_{yj}(s),\ s\in[-\tau,0],\ j=1,2,\cdots,m,
\end{cases}
$$

其中 $\varphi_{xi}(s)$,$\varphi_{yj}(s)$ 分别表示定义在 $[-\sigma,0]$,$[-\tau,0]$ 上的实值连续函数.

1 预备知识

假设

(H_1) $\forall x,y\in\mathbf{R}$,$x\neq y$,$i=1,2,\cdots,n$,$j=1,2,\cdots,m$,存在实数 $F_i>0$,$G_j>0$,有

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$|f_i(x)| \leqslant F_i, |g_j(x)| \leqslant G_j$;

(H_2) $\forall x, y \in \mathbf{R}, x \neq y$, $|f_j(x) - f_j(y)| \leqslant L_j|x-y|, |g_i(x) - g_i(y)| \leqslant M_i|x-y|$;

(H_3) $\underline{a}_i = \min_{t\in[0,\omega]} a_i(t), \underline{b}_j = \min_{t\in[0,\omega]} b_j(t), \bar{c}_{ij} = \max_{t\in[0,\omega]} c_{ij}(t), \bar{d}_{ji} = \max_{t\in[0,\omega]} d_{ji}(t), \bar{e}_{ijl} = \max_{t\in[0,\omega]} e_{ijl}(t), \bar{\omega}_{jil} = \max_{t\in[0,\omega]} \omega_{jil}(t), \bar{e}_{ilj} = \max_{t\in[0,\omega]} e_{ilj}(t), \bar{\omega}_{jli} = \max_{t\in[0,\omega]} \omega_{jli}(t)$.

定义范数: 对于系统(1)的任意解和周期解

$\boldsymbol{x}(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$,

$\boldsymbol{y}(t) = (y_1(t), y_2(t), \cdots, y_m(t))^{\mathrm{T}}$,

$\boldsymbol{x}^*(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$,

$\boldsymbol{y}^*(t) = (y_1(t), y_2(t), \cdots, y_m(t))^{\mathrm{T}}$,

$$\|\boldsymbol{x}(t) - \boldsymbol{x}^*(t)\|_r = \left[\sum_{i=1}^{n} |x_i(t) - x_i^*(t)|^r\right]^{1/r},$$

$$\|\boldsymbol{\varphi}_x(s) - \boldsymbol{x}^*(t)\|_r = \left[\sum_{i=1}^{n} \sup_{s\in[-\sigma,0]} |\varphi_{xi}(s) - x_i^*(t)|^r\right]^{1/r},$$

$$\|\boldsymbol{y}(t) - \boldsymbol{y}^*(t)\|_r = \left[\sum_{j=1}^{m} |y_j(t) - y_j^*(t)|^r\right]^{1/r},$$

$$\|\boldsymbol{\varphi}_y(s) - \boldsymbol{y}^*(t)\|_r = \left[\sum_{j=1}^{m} \sup_{s\in[-\tau,0]} |\varphi_{yj}(s) - y_j^*(t)|^r\right]^{1/r},$$

其中 $r > 1$.

定义 1[7] 若存在常数 $\kappa \geqslant 1$ 和 $\hat{\varepsilon} > 0$ 满足

$\|\boldsymbol{x}(t) - \boldsymbol{x}^*(t)\|_r + \|\boldsymbol{y}(t) - \boldsymbol{y}^*(t)\|_r \leqslant \kappa(\|\boldsymbol{\varphi}_x(s) - \boldsymbol{x}^*(t)\|_r + \|\boldsymbol{\varphi}_y(s) - \boldsymbol{y}^*(t)\|_r)\mathrm{e}^{-\hat{\varepsilon}t}$,

则系统的周期解全局指数稳定.

定义 2 若实矩阵 $\boldsymbol{A} = (a_{ij})_{n\times n}$ 满足 $a_{ii} > 0(i = 1,2,\cdots,n)$, $a_{ij} \leqslant 0(i \neq j, i = 1,2,\cdots,n)$, 且 $\boldsymbol{A}^{-1} \geqslant 0$, 则称 $\boldsymbol{A}$ 为 M-矩阵.

引理 1[7] 若实矩阵 $\boldsymbol{A}$ 为 M-矩阵, 则

(i) 存在一个向量 $\boldsymbol{\eta} = (\eta_1, \eta_2, \cdots, \eta_n) > 0$, 使得 $\boldsymbol{A\eta} > 0$;

(ii) 存在一个向量 $\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_n) > 0$, 使得 $\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{A} > 0$.

引理 2[7] 设 $a \geqslant 0, b \geqslant 0, p > 1$, 则 $pa^{p-1}b \leqslant (p-1)a^p + b^p$.

2 主要结论及其证明

定理 1 若系统(1)满足$(H_1) \sim (H_3)$, 且矩阵 $\boldsymbol{A}$ 是一个 M-矩阵,

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{A}_3 & \boldsymbol{A}_4 \end{pmatrix},$$

其中

$\boldsymbol{A}_1 = \mathrm{diag}(r\underline{a}_1 - (r-1)\sum_{j=1}^{m}|\bar{c}_{1j}| - (r-1) \cdot \sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{1jl} + \bar{e}_{1lj}|F_l, \cdots, r\underline{a}_n - (r-1)\sum_{j=1}^{m}|\bar{c}_{nj}| - \sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{njl} + \bar{e}_{nlj}|F_l)$,

$\boldsymbol{A}_2 = (A_{ij}^{(2)})_{n\times m}, A_{ij}^{(2)} = -(|\bar{c}_{ij}| + \sum_{l=1}^{m} F_l|\bar{e}_{ijl} + \bar{e}_{ilj}|)L_j^r$,

$\boldsymbol{A}_3 = (A_{ji}^{(3)})_{m\times n}, A_{ji}^{(3)} = -(|\bar{d}_{ji}| + \sum_{l=1}^{n} G_l|\bar{\omega}_{jil} + \bar{\omega}_{jli}|)M_i^r$,

$\boldsymbol{A}_4 = \mathrm{diag}(r\underline{b}_1 - (r-1)\sum_{i=1}^{n}|\bar{d}_{1i}| - (r-1) \cdot \sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{1il} + \bar{\omega}_{1li}|G_l, \cdots, r\underline{b}_m - (r-1)\sum_{i=1}^{n}|\bar{d}_{mi}| - (r-1)\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{mil} + \bar{\omega}_{mli}|G_l)$,

这里 $r > 1$, 则系统(1)的周期解全局指数稳定.

证 设 $(\boldsymbol{x}(t), \boldsymbol{y}(t))^{\mathrm{T}} = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_m(t))^{\mathrm{T}}$ 是系统(1)的任意解, $(\boldsymbol{x}^*(t), \boldsymbol{y}^*(t))^{\mathrm{T}} = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t), y_1^*(t), y_2^*(t), \cdots, y_n^*(t))^{\mathrm{T}}$ 是系统(1)的一个周期解, $u_i(t) = x_i(t) - x_i^*(t)$, $v_j(t) = y_j(t) - y_j^*(t)$, $\tilde{f}_j(v_j(t)) = f_j(v_j(t) + y_j^*(t)) - f_j(y_j^*(t))$, $\tilde{g}_j(u_i(t)) = g_i(u_i(t) + x_i^*(t)) - g_i(x_i^*(t))$, 则

$$\mathrm{d}|u_i(t)|/\mathrm{d}t = -a_i(t)|u_i(t)| + \sum_{j=1}^{m}|c_{ij}(t)| \cdot |\tilde{f}_j(v_j(t-\tau_j))| + \sum_{j=1}^{m}\sum_{l=1}^{m}|e_{ijl}(t)||[f_j(y_j(t-\tau_j)) - f_j(y_j^*(t)))f_l(y_l(t-\tau_l)) + (f_l(y_l(t-\tau_l)) - f_l(y_l^*(t)))f_j(y_j^*(t))]| = -a_i(t)|u_i(t)| + \sum_{j=1}^{m}|c_{ij}(t)||\tilde{f}_j(v_j(t-\tau_j))| + \sum_{j=1}^{m}\sum_{l=1}^{m}|e_{ijl}(t)| \cdot |[\tilde{f}_j(v_j(t-\tau_j))f_l(y_l(t-\tau_l)) + \tilde{f}_l(v_l(t-\tau_l)f_j(y_j^*(t)]| = -a_i(t)|u_i(t)| + \sum_{j=1}^{m}[|c_{ij}(t)| + \sum_{l=1}^{m}|e_{ijl}(t)||f_l(y_l(t-\tau_l))| + |e_{ilj}(t)||f_l(y_l^*(t))|]|\tilde{f}_j(v_j(t-\tau_j))| = -a_i(t)|u_i(t)| + \sum_{j=1}^{m}[|c_{ij}(t)| + \sum_{l=1}^{m}[|e_{ijl}(t)| + e_{ilj}(t)||\xi_l(t)|]]|\tilde{f}_j(v_j(t-\tau_j))|, t \geqslant 0, t \neq t_k.$$

同理可得

$$\mathrm{d}|v_j(t)|/\mathrm{d}t = -b_j(t)|v_j(t)| + \sum_{i=1}^{n}[|d_{ji}(t)| + \sum_{l=1}^{n}|\omega_{jil}(t) + \omega_{jli}(t)||\eta_l(t)|] \cdot |\tilde{g}_j(u_i(t-\sigma_i))|, t \geqslant 0, t \neq t_k,$$

其中

$$\xi_l(t)=\begin{cases}\dfrac{e_{ijl}(t)}{e_{ijl}(t)+e_{ilj}(t)}f_l(y_l(t-\tau_l))+\\ \dfrac{e_{ijl}(t)}{e_{ijl}(t)+e_{ilj}(t)}f_l(y_l^*(t)),e_{ijl}(t)+\\ e_{ilj}(t)\neq 0,\\ 0,e_{ijl}(t)+e_{ilj}(t)=0,\end{cases}$$

$$\eta_l(t)=\begin{cases}\dfrac{\omega_{jil}(t)}{\omega_{jil}(t)+\omega_{jli}(t)}g_l(x_l(t-\sigma_l))+\\ \dfrac{\omega_{jil}(t)}{\omega_{jil}(t)+\omega_{jli}(t)}g_l(x_i^*(t)),\omega_{jil}(t)+\\ \omega_{jli}(t)\neq 0,\\ 0,\omega_{jil}(t)+\omega_{jli}(t)=0.\end{cases}$$

$$|u_i(t_k+0)|=|x_i(t_k+0)-x_i^*(t_k+0)|=|(1-\gamma_{ik})||x_i(t_k)-x_i^*(t_k)|\leqslant|u_i(t_k)|,$$
$$|v_j(t_k+0)|=|y_j(t_k+0)-y_j^*(t_k+0)|=|(1-\beta_{jk})||y_j(t_k)-y_j^*(t_k)|\leqslant|v_j(t_k)|.$$

由 A 是 M-矩阵和引理 1 知，必存在向量 $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\cdots,\lambda_{n+m})^{\mathrm{T}}>0$ 满足

$$\lambda_i\Big[\underline{a}_i-(r-1)\sum_{j=1}^m|\bar c_{ij}|-(r-1)\sum_{j=1}^m\sum_{l=1}^m|\bar e_{ijl}+\bar e_{ilj}|F_l\Big]-\sum_{j=1}^m\lambda_{n+j}\Big[|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big]M_i^r>0,$$

$$\lambda_{n+j}\Big[\underline{b}_j-(r-1)\sum_{i=1}^n|\bar d_{ji}|-(r-1)\sum_{i=1}^n\sum_{l=1}^n|\bar\omega_{jil}+\bar\omega_{jli}|G_l\Big]-\sum_{i=1}^n\lambda_i\Big[|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big]L_j^r>0,$$

这里将 H_i 和 E_j 定义为

$$H_i(\theta)=\lambda_i\Big[\theta-r\underline{a}_i+(r-1)\sum_{j=1}^m|\bar c_{ij}|+(r-1)\sum_{j=1}^m\sum_{l=1}^m|\bar e_{ijl}+\bar e_{ilj}|F_l\Big]+\sum_{j=1}^m\lambda_{n+j}\Big[|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big]\mathrm{e}^{\theta\sigma_i}M_i^r,$$

$$E_j(\theta)=\lambda_{n+j}\Big[\theta-r\underline{b}_j+(r-1)\sum_{i=1}^n|\bar d_{ji}|+(r-1)\sum_{i=1}^n\sum_{l=1}^n|\bar\omega_{jil}+\bar\omega_{jli}|G_l\Big]+\sum_{i=1}^n\lambda_i\Big[|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big]\mathrm{e}^{\theta\tau_j}L_j^r,$$

由上述两式可知，$H_i(0)<0$，$E_j(0)<0$，当 $\theta\to+\infty$ 时，$H_i(\theta)$，$E_j(\theta)\to+\infty$，$\mathrm{d}H_i(\theta)/\mathrm{d}\theta>0$，$\mathrm{d}E_j(\theta)/\mathrm{d}\theta>0$，$H_i(\theta)$ 和 $E_j(\theta)$ 在 $[0,+\infty)$ 上严格单调递增. 因此存在 ζ_i^*，ξ_j^* 满足

$$H_i(\zeta_i^*)=\lambda_i\Big[\zeta_i^*-r\underline{a}_i+(r-1)\sum_{j=1}^m|\bar c_{ij}|+(r-1)\cdot\sum_{j=1}^m\sum_{l=1}^m|\bar e_{ijl}+\bar e_{ilj}|F_l\Big]+\sum_{j=1}^m\lambda_{n+j}\Big[|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big]\mathrm{e}^{\zeta_i^*\sigma_i}M_i^r=0,$$

$$E_j(\xi_j^*)=\lambda_{n+j}\Big[\xi_j^*-r\underline{b}_j+(r-1)\sum_{i=1}^n|\bar d_{ji}|+(r-1)\cdot\sum_{i=1}^n\sum_{l=1}^n|\bar\omega_{jil}+\bar\omega_{jli}|G_l\Big]+\sum_{i=1}^n\lambda_i\Big[|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big]\mathrm{e}^{\xi_j^*\tau_i}L_j^r=0,$$

因此可取 $0<\varepsilon<\min\{\zeta_1^*,\zeta_2^*,\cdots,\zeta_n^*,\xi_1^*,\xi_2^*,\cdots,\xi_m^*\}$，使得

$$H_i(\varepsilon)=\lambda_i\Big[\varepsilon-r\underline{a}_i+(r-1)\sum_{j=1}^m|\bar c_{ij}|+(r-1)\cdot\sum_{j=1}^m\sum_{l=1}^m|\bar e_{ijl}+\bar e_{ilj}|F_l\Big]+\sum_{j=1}^m\lambda_{n+j}\Big[|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big]\mathrm{e}^{\varepsilon\sigma_i}M_i^r<0,$$

$$E_j(\varepsilon)=\lambda_{n+j}\Big[\varepsilon-r\underline{b}_j+(r-1)\sum_{i=1}^n|\bar d_{ji}|+(r-1)\cdot\sum_{i=1}^n\sum_{l=1}^n|\bar\omega_{jil}+\bar\omega_{jli}|G_l\Big]+\sum_{i=1}^n\lambda_i\Big[|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big]\mathrm{e}^{\varepsilon\tau_i}L_j^r<0. \qquad (2)$$

定义李雅普诺夫函数

$$\tilde v(t)=\sum_{i=1}^n\lambda_i\Big\{\mathrm{e}^{\varepsilon t}|u_i(t)|^r+\sum_{j=1}^m\Big[|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big]\int_{t-\tau_j}^t\mathrm{e}^{\varepsilon(s+\tau_j)}|\tilde f_j(v_j(s))|^r\mathrm{d}s\Big\}+\sum_{j=1}^m\lambda_{n+j}\Big\{\mathrm{e}^{\varepsilon t}|v_j(t)|^r+\sum_{i=1}^n\Big[|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big]\int_{t-\sigma_i}^t\mathrm{e}^{\varepsilon(s+\sigma_i)}|\tilde g_i(u_i(s))|^r\mathrm{d}s\Big\},$$

由引理 2 和(2)式可得

$$D^+\tilde v(t)=\sum_{i=1}^n\lambda_i\Big\{\varepsilon\mathrm{e}^{\varepsilon t}|u_i(t)|^r+r\mathrm{e}^{\varepsilon t}|u_i(t)|^{r-1}\cdot\Big[-a_i(t)|u_i(t)|+\sum_{j=1}^m\Big[|c_{ij}(t)|+\sum_{l=1}^m\Big[|e_{ijl}(t)+e_{ilj}(t)||\xi_l(t)|\Big]\Big]\Big]|\tilde f_j(v_j(t-\tau_j))|+\sum_{j=1}^m\Big(|\bar c_{ij}|+\sum_{l=1}^m F_l|\bar e_{ijl}+\bar e_{ilj}|\Big)\Big[\mathrm{e}^{\varepsilon(t+\tau_j)}|\tilde f_j(v_j(t))|^r-\mathrm{e}^{\varepsilon t}|\tilde f_j(v_j(t-\tau_j))|^r\Big]\Big\}+\sum_{j=1}^m\lambda_{n+j}\Big\{\varepsilon\mathrm{e}^{\varepsilon t}|v_j(t)|^r+r\mathrm{e}^{\varepsilon t}|v_j(t)|^{r-1}\cdot\Big[-b_j(t)|v_j(t)|+\sum_{j=1}^m\Big[d_{ji}(t)+\sum_{l=1}^m\Big[|\omega_{jil}(t)+\omega_{jli}(t)||\eta_l(t)|\Big]\Big]\Big]|\tilde g_i(u_i(t-\sigma_i))|+\sum_{i=1}^n\Big(|\bar d_{ji}|+\sum_{l=1}^n G_l|\bar\omega_{jil}+\bar\omega_{jli}|\Big)\Big[\mathrm{e}^{\varepsilon(t+\sigma_i)}|\tilde g_i(u_i(t))|^r-\mathrm{e}^{\varepsilon t}|\tilde g_i(u_i(t-$$

$$\sigma_i))|^r]\} \leqslant \sum_{i=1}^{n}\lambda_i\{\varepsilon e^{\varepsilon t}|u_i(t)|^r - r e^{\varepsilon t}\underline{a}_i|u_i(t)|^r + e^{\varepsilon t}\sum_{j=1}^{m}|\bar{c}_{ij}|[(r-1)|u_i(t)|^r + |\tilde{f}_j(v_j(t-\tau_j))|^r] + e^{\varepsilon t}\sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{ijl}+\bar{e}_{ilj}|F_l[(r-1)|u_i(t)|^r + |\tilde{f}_j(v_j(t-\tau_j))|^r] + e^{\varepsilon t}\sum_{j=1}^{m}(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)[e^{\varepsilon\tau_j}|\tilde{f}_j(v_j(t))|^r - |\tilde{f}_j(v_j(t-\tau_j))|^r]\} +$$

$$\sum_{j=1}^{m}\lambda_{n+j}\{\varepsilon e^{\varepsilon t}|v_j(t)|^r - r e^{\varepsilon t}\underline{b}_j|v_j(t)|^r + e^{\varepsilon t}\sum_{i=1}^{n}|\bar{d}_{ji}|[(r-1)|v_j(t)|^r + |\tilde{g}_i(u_i(t-\sigma_i))|^r] + e^{\varepsilon t}\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{jil}+\bar{\omega}_{jli}|G_l[(r-1)|v_j(t)|^r + |\tilde{g}_i(u_i(t-\sigma_i))|^r] + e^{\varepsilon t}\sum_{i=1}^{n}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)[e^{\varepsilon\sigma_i}|\tilde{g}_i(u_i(t))|^r - |\tilde{g}_i(u_i(t-\sigma_i))|^r]\} =$$

$$\sum_{i=1}^{n}\lambda_i e^{\varepsilon t}[\varepsilon - r\underline{a}_i + (r-1)\sum_{j=1}^{m}|\bar{c}_{ij}| + (r-1)\sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{ijl}+\bar{e}_{ilj}|F_l]|u_i(t)|^r + \sum_{j=1}^{m}(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)e^{\varepsilon\tau_j}|\tilde{f}_j(v_j(t))|^r] +$$

$$\sum_{j=1}^{m}\lambda_{n+j}e^{\varepsilon t}[\varepsilon - r\underline{d}_j + (r-1)\sum_{i=1}^{n}|\bar{d}_{ji}| + (r-1)\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{jil}+\bar{\omega}_{jli}|G_l]|v_j(t)|^r + \sum_{i=1}^{n}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)e^{\varepsilon\sigma_i}|\tilde{g}_i(u_i(t))|^r] \leqslant$$

$$\sum_{i=1}^{n}[\lambda_i(\varepsilon - r\underline{a}_i + (r-1)\sum_{j=1}^{m}|\bar{c}_{ij}| + (r-1)\sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{ijl}+\bar{e}_{ilj}|F_l) + \sum_{i=1}^{n}\lambda_{n+j}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)e^{\varepsilon\sigma_i}M_i^r]\cdot e^{\varepsilon t}|u_i(t)|^r +$$

$$\sum_{j=1}^{m}[\lambda_{n+j}(\varepsilon - r\underline{b}_j + (r-1)\sum_{i=1}^{n}|\bar{d}_{ji}| + (r-1)\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{jil}+\bar{\omega}_{jli}|G_l) + \sum_{i=1}^{n}\lambda_i(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)e^{\varepsilon\tau_j}L_j^r]e^{\varepsilon t}|v_j(t)|^r \leqslant 0,$$

这里 $t>0, t\neq t_k, k=1,2,\cdots$,所以,$\tilde{v}(t)\leqslant\tilde{v}(t_k+0), t\in(t_k,t_{k+1}], k=1,2,\cdots$. 由李雅普诺夫函数知,$\tilde{v}(t_k+0)\leqslant\tilde{v}(t_k)$,则 $\tilde{v}(t)\leqslant\tilde{v}(0), t>0$,且

$$\tilde{v}(0) = \sum_{i=1}^{n}\lambda_i\{e^{\varepsilon t}|u_i(0)|^r + \sum_{j=1}^{m}[(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|]\int_{-\tau_j}^{0}e^{\varepsilon(s+\tau_j)}|\tilde{f}_j(v_j(s))|^r ds\} + \sum_{j=1}^{m}\lambda_{n+j}\{e^{\varepsilon t}|v_j(0)|^r + \sum_{i=1}^{n}[|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|]\cdot\int_{-\sigma_i}^{0}e^{\varepsilon(s+\sigma_i)}|\tilde{g}_i(u_i(s))|^r ds\} \leqslant$$

$$\bar{\lambda}\{\sum_{i=1}^{n}[|u_i(0)|^r + \sum_{j=1}^{m}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)e^{\varepsilon\sigma}M_i^r\int_{-\sigma_i}^{0}|u_i(s)|^r ds] + \sum_{j=1}^{m}[|v_j(0)|^r + \sum_{i=1}^{n}(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)e^{\varepsilon\tau}L_j^r\cdot\int_{-\tau_i}^{0}|v_j(s)|^r ds]\},$$

$$\tilde{v}(t) \geqslant \underline{\lambda}e^{\varepsilon t}[\sum_{i=1}^{n}|u_i(t)|^r + \sum_{j=1}^{m}|v_j(t)|^r],$$

其中 $\bar{\lambda}=\max\{r_1,r_2,\cdots,r_{n+m}\}$, $\underline{\lambda}=\min\{r_1,r_2,\cdots,r_{n+m}\}$,

$$\sum_{i=1}^{n}|u_i(t)|^r + \sum_{j=1}^{m}|v_j(t)|^r \leqslant e^{-\varepsilon t}\frac{\bar{\lambda}}{\underline{\lambda}}\{\sum_{i=1}^{n}[|u_i(0)|^r + \sum_{j=1}^{m}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)e^{\varepsilon\sigma}M_i^r\sigma\sup_{s\in[-\sigma,0]}|u_i(s)|^r] + \sum_{j=1}^{m}[|v_j(0)|^r + \sum_{i=1}^{n}((|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)e^{\varepsilon\tau}L_j^r\tau\cdot\sup_{s\in[-\tau,0]}|v_j(s)|^r]\}.$$

由上式及定义1可知

$$\|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\|_r^r + \|\boldsymbol{y}(t)-\boldsymbol{y}^*(t)\|_r^r = \sum_{i=1}^{n}|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)|^r + \sum_{j=1}^{m}|\boldsymbol{y}(t)-\boldsymbol{y}^*(t)|^r \leqslant$$

$$e^{-\varepsilon t}\frac{\bar{\lambda}}{\underline{\lambda}}\{\sum_{i=1}^{n}[1+\sigma\max_{1\leqslant i\leqslant n}(\sum_{j=1}^{m}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)M_i^r)e^{\varepsilon\sigma}]\sup_{s\in[-\sigma,0]}|x_i(s)-x_i^*(s)|^r + \sum_{j=1}^{m}[1+\tau\max_{1\leqslant j\leqslant m}(\sum_{i=1}^{n}(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)L_j^r)e^{\varepsilon\tau}]\cdot\sup_{s\in[-\tau,0]}|y_j(s)-y_j^*(s)|^r \leqslant ke^{-\varepsilon t}[\sum_{i=1}^{n}\sup_{s\in[-\sigma,0]}|x_i(s)-x_i^*(s)|^r + \sum_{j=1}^{m}\sup_{s\in[-\tau,0]}|y_j(s)-y_j^*(s)|^r] = k(\|\boldsymbol{\varphi}_x(s)-\boldsymbol{x}^*(t)\|_r^r + \|\boldsymbol{\varphi}_y(s)-\boldsymbol{y}^*(t)\|_r^r)e^{-\varepsilon t},$$

其中 $k=\max\frac{\bar{\lambda}}{\underline{\lambda}}\{1+\sigma\max_{1\leqslant i\leqslant n}(\sum_{j=1}^{m}(|\bar{d}_{ji}| + \sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)M_i^r)e^{\varepsilon\sigma}, 1+\tau\max_{1\leqslant j\leqslant m}(\sum_{i=1}^{n}(|\bar{c}_{ij}| + \sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)\cdot L_j^r)e^{\varepsilon\tau}\}$.

由文献[7]可知,

$$\|\boldsymbol{x}(t)-\boldsymbol{x}^*(t)\|_r + \|\boldsymbol{y}(t)-\boldsymbol{y}^*(t)\|_r \leqslant \kappa(\|\boldsymbol{\varphi}_x(s)-\boldsymbol{x}^*(t)\|_r + \|\boldsymbol{\varphi}_y(s)-\boldsymbol{y}^*(t)\|_r)e^{-\hat{\varepsilon}t},$$

定理1得证.

推论1 若系统(1)满足(H_1)~(H_3),且矩阵 $\boldsymbol{A}$ 是一个 M-矩阵,其中

$$\boldsymbol{A} = \begin{pmatrix}\boldsymbol{A}_1 & \boldsymbol{A}_2\\ \boldsymbol{A}_3 & \boldsymbol{A}_4\end{pmatrix},$$

$$A_1=\mathrm{diag}(2\underline{a}_1-\sum_{j=1}^{m}|\bar{c}_{1j}|-\sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{1jl}+\bar{e}_{1lj}|F_l,\cdots,2\underline{a}_n-\sum_{j=1}^{m}|\bar{c}_{nj}|-\sum_{j=1}^{m}\sum_{l=1}^{m}|\bar{e}_{njl}+\bar{e}_{nlj}|F_l),$$

$$A_2=(A_{ij}^{(2)})_{n\times m},A_{ij}^{(2)}=-(|\bar{c}_{ij}|+\sum_{l=1}^{m}F_l|\bar{e}_{ijl}+\bar{e}_{ilj}|)L_j^2,$$

$$A_3=(A_{ji}^{(3)})_{m\times n},A_{ji}^{(3)}=-(|\bar{d}_{ji}|+\sum_{l=1}^{n}G_l|\bar{\omega}_{jil}+\bar{\omega}_{jli}|)M_i^2,$$

$$A_4=\mathrm{diag}(2\underline{b}_1-\sum_{i=1}^{n}|\bar{d}_{1i}|-\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{1il}+\bar{\omega}_{1li}|G_l,\cdots,2\underline{b}_m-\sum_{i=1}^{n}|\bar{d}_{mi}|-\sum_{i=1}^{n}\sum_{l=1}^{n}|\bar{\omega}_{mil}+\bar{\omega}_{mli}|G_l),$$

则系统(1)的周期解全局指数稳定.

证　令 $r=2$，由定理 1 可得推论 1.

注 1　推论 1 是文献[11]中的一个定理，因此与以往文献相比，本文结果更具一般性.

下面举例说明定理 1 的应用.

例 1　考虑系统

$$\begin{cases}\mathrm{d}x_1(t)/\mathrm{d}t=-a_1(t)x_1(t)+c_{11}(t)f_1(y_1(t-\tau_1))+e_{111}(t)(f_1(y_1(t-\tau_1)))^2+I_1(t),\\ \Delta x_1(t_k)=-\gamma_{1k}x_1(t_k),k=1,2,\cdots,\\ \mathrm{d}y_1(t)/\mathrm{d}t=-b_1(t)y_1(t)+d_{11}(t)g_1(x_1(t-\sigma_1))+\omega_{111}(t)(g_1(x_1(t-\sigma_1)))^2+J_1(t),\\ \Delta y_1(t_k)=-\beta_{1k}y_1(t_k),k=1,2,\cdots.\end{cases}\quad(3)$$

$f_1(y_1(t))=0.1\tanh(y_1(t))$，$g_1(x_1(t))=0.2\tanh(x_1(t))$，$a_1(t)=b_1(t)=1$，$c_{11}(t)=-0.3\sin t$，$d_{11}(t)=-\cos t$，$\tau_1=\sigma_1=0.5$，$e_{111}(t)=\omega_{111}(t)=\cos t$，$I_1(t)=\sin t$，$\gamma_{1k}=\beta_{1k}=1$，$t_k=0.4\pi k$，$J_1(t)=\cos t$，$G_1=M_1=0.2$，$F_1=L_1=0.1$，由推论 1 可知，

$$A=\begin{pmatrix}1.300&-0.007\\-0.048&0.800\end{pmatrix}$$

是 M-矩阵，因此系统(3)的周期解是全局指数稳定的.

3　结论

针对一类含脉冲的非自治高阶 BAM 神经网络，基于 M-矩阵理论以及 Yang 不等式技巧，导出了其周期解的全局指数稳定性的充分条件，该结果与以往文献相比，更具一般性，对于高阶 BAM 神经网络的设计与应用更具有一定的理论和现实意义.

4　参考文献

[1] Kosko B. Bi-dierctional associative memories [J]. IEEE Transanction System Man Cybernet，1988，18(1)：49-60.

[2] Cao Jinde，Meng Dongfeng. Exponential stability of delayed bidirectional associative memory neural networks [J]. Applied Mathematics and Computation，2003，135(3)：102-117.

[3] Cao Jinde. Global asymptotic stability of delayed bi-directional associative memory neural networks [J]. Applied Mathematics and Computation，2003，142(2/3)：333-339.

[4] Cao Jinde，Liang Jinling. Global asytmptotic stability of bi-directional associative memory neural networks with distributed delays [J]. Applied Mathematics and Computation，2004，152(2)：415-424.

[5] Li Yongkun. Global exponential stability of BAM neural networks with delays and impulses [J]. Chaos，Solitons and Fractals，2005，24(1)：279-285.

[6] Li Yaotang，Yang Changbo. Global exponential stability analysis on impulisive BAM neural networks with distributed delays [J]. Journal of Mathematical Analysis and Applications，2006，324(2)：1125-1139.

[7] Li Yaotang，Wang Jiyu. An analysis on global exponential stability and the existence of periodic solutions for non-autonomous hybrid BAM neural networks with distributed delays and impulses [J]. Computers and Mathematics with Applications，2008，56(9)：2256-2267.

[8] Wang Yangling，Cao Jinde. Exponential stability of stochastic higher-order BAM neural networks with reaction-diffusion terms and mixed time-varying delays [J]. Neurocomputing，2013，119(16)：192-200.

[9] Zhang Ancai，Qiu Jianlong，She Jinhua. Existence and global exponential stability of periodic solution for high-order discrete-time BAM neural networks [J]. Neural Networks，2014，50(2)：98-109.

[10] Xu Changjin，Zhang Qiming. Existence and global exponential stability of anti-periodic solutions of high-order bidirectional associative memory(BAM) networks with time-varying delays on time scales [J]. Journal of Computational Science，2015，8：48-61.

[11] Wang Fen，Sun Dong，Wu Huaiyu. Global exponential stability and periodic solutions of high-order bidirectional associative memory(BAM) neural networks with time delays and impulses [J]. Neurocomputing，2015，155(3)：261-276.

[12] Gu Haibo. Mean square exponential stability in high-order stochastic impulsive BAM neural networks with time-varying delays [J]. Neurocomputing，2011，74(5)：720-729.

[13] Li Yongkun，Li Yang. Almost automorphic solution for neutral type high-order Hopfield neural networks with delays in leakage terms on time scales [J]. Applied Mathematics and Computation，2014，242：679-693.

[14] Song Qiankun，Cao Jinde. Dynamical behaviors of discrete-time fuzzy cellular neural networks with variable delays

and impulse [J]. Journal of the Franklin Institute, 2008, 345(1): 39-59.

[15] Li Yongkun, Xing Zhiwei. Existence and global exponential stability of periodic solutions for CNNS with impulse [J]. Chaos, Solitons and Fractals, 2007, 33(5): 1686-1693.

The Analysis on Global Exponential Stability of Periodic Solutions for a Class of Non-Autonomous Higher-Order BAM Neural Networks with Impulse

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Abstract: By constructing a new Lyapunov functional, employing the M-matrix theory and some inequality techniques, the global exponential stability of periodic solutions for non-autonomous higher-order BAM neural networks with impulse is considered, and it has greatly improve the previous results in the literature.

Key words: higher-order BAM neural networks; periodic solutions; M-matrix; impulse; exponential stability

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The Stability of Motion for the Generalized Birkhoffian System with Constrains

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Abstract: The problem on the stability of motion for a generalized Birkhoffian system with constrains are studied by the Noether theong. The disturbed equations of motion and their first approximation for the system are established. The criterion of stability of motion for the system was set up by using Lyapnnov´s first approximation theory. The Lyapnnov´s function was constructed by the Noether conserved quantity and the criterion of stability of motion for the system was also set up by using Lyapnnov´s direct method. Finally, the example is given to illustrate the application of the results.

Key words: generalized Birkhoffian system; stability of motion; first approximation theory; direct method

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