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一类含η-次可微映射的广义拟-似变分包含组

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摘要: 使用 η-近似映射技巧, 证明一类含 η-次可微映射的广义拟-似变分包含组解的存在性和 1 个 N-步迭代算法的收敛性, 改进和推广了近期一些熟知的结果.

关键词: 广义拟-似变分包含组; η-近似映射; 单调映射; 迭代算法
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0　引　言

本文总设 H 为Hilbert空间, $CB(H)$ 为 H 中不空有界闭子集全体.

考察如下问题: 对 $i=1,2,\cdots,N$, 设 $A_i:H\to CB(H)$, $\eta_i:H\times H\to H$, $g_i:H\to H$, $T_i:\underbrace{H\times H\times\cdots\times H}_{N个}\to H$ 是非线性映射, $\varphi_i:H\to \mathbf{R}\cup\{+\infty\}$ 是真泛函, 找 $x_1^*,x_2^*,\cdots,x_N^*\in H$, $u_1^*\in A_1x_1^*$, $u_2^*\in A_2x_2^*,\cdots,$ $u_N^*\in A_Nx_N^*$ 使得

$$\begin{cases}\left\langle T_1(u_1^*,u_2^*,\cdots,u_N^*),\ \eta_1(x,g_1(x_1^*))\right\rangle\geqslant\varphi_1(g_1(x_1^*))-\varphi_1(x),\\ \left\langle T_2(u_1^*,u_2^*,\cdots,u_N^*),\ \eta_2(x,g_2(x_2^*))\right\rangle\geqslant\varphi_2(g_2(x_2^*))-\varphi_2(x),\\ \qquad\vdots\\ \left\langle T_N(u_1^*,u_2^*,\cdots,u_N^*),\ \eta_N(x,g_N(x_N^*))\right\rangle\geqslant\\ \varphi_N(g_N(x_N^*))-\varphi_N(x),\quad\forall x\in H,\end{cases}\tag{1}$$

称之为集值非线性广义拟-似变分包含组.

特殊情况:

(i) 如果 $N=1, A_1=A,\ A_2=B,\ T_1(A(\cdot),B(\cdot)):H\to CB(H)$, 则(1)式归结为找 $x^*\in H$, $u^*\in Ax^*$, $v^*\in Bx^*$, 使得

$$\left\langle T_1(u^*,v^*),\ \eta(x,g(x^*))\right\rangle\geqslant\varphi(g(x^*))-\varphi(x),\quad\forall x\in H.$$

它在 2001 年由 Ding Xie-ping[1]引进和研究.

(ii) 如果 $N=2, A_i=I, g_i=I$(恒同算子), $\eta(x,y)=x-y,\ \varphi_1=\varphi_2=\varphi,\ T_1(x,y)=T(y,x)+x-y,\ T_2(x,y)=T(x,y)+y-x$, 则(1)式归结为找 $x^*,y^*\in H$ 使得

$$\begin{cases}\left\langle\rho T(y^*,x^*)+x^*-y^*,x-x^*\right\rangle+\varphi(x)-\\ \varphi(x^*)\geqslant0,\ \forall x\in H,\rho>0,\\ \left\langle\eta T(x^*,y^*)+y^*-x^*,x-y^*\right\rangle+\varphi(x)-\\ \varphi(y^*)\geqslant0,\ \forall x\in H,\eta>0.\end{cases}\tag{2}$$

它由He Zhen-hua等在文献[2]中研究(包括 $\rho=\eta=1$ 情况).

(iii) 设 $K\subset H$ 为闭凸子集, $\varphi(x)=I_K(x)$, 则(2)式等价于找 $x^*,y^*\in K$ 使得

$$\begin{cases}\left\langle\rho T(y^*,x^*)+x^*-y^*,x-x^*\right\rangle\geqslant0,\ \forall x\in K,\rho>0,\\ \left\langle\eta T(x^*,y^*)+y^*-x^*,x-y^*\right\rangle\geqslant0,\ \forall x\in K,\eta>0.\end{cases}\tag{3}$$

它由S.S.Chang[3], R.U.Verma[4-5]及Huang Zhen-yu[6]考察和研究过.

(iv) 如果 $N=2,T_1(x,y)=\rho Ty+g(x)-g(y),T_2(x,y)=\eta Tx+g(y)-g(x),\ \eta(x,y)=g(x)-g(y),\rho,\eta>0$, 则(1)式归结为找 $x^*,y^*\in H$ 使得

$$\begin{cases}\left\langle\rho Ty^*+g(x^*)-g(y^*),g(x)-g(x^*)\right\rangle\geqslant0,\ \forall x\in H,\\ \left\langle\eta Tx^*+g(y^*)-g(x^*),g(x)-g(y^*)\right\rangle\geqslant0,\ \forall x\in H.\end{cases}$$

它由A. Hajjafar等[7]引进和研究.

更多的情况参看文献[1-8].

2006年, Yang在文献[9]中指出了文献[5]中存在的1个问题, 如果问题(3)中的解 $x^*\neq y^*$, 则当取 $x=y^*$ 时, 问题(3)的第1式有

$$\left\langle\rho T(y^*,x^*),y^*-x^*\right\rangle\geqslant\left\|x^*-y^*\right\|.$$

显然, 当 $\rho\to0$ 时, 上式只有当 $x^*=y^*$ 时才成

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立. 此时，就退化成1个不等式方程.

本文的目的是研究问题(1)，用η-近似映射技巧，证明了这个问题解的存在性和1个N-步迭代算法的收敛性.结果是文献[1-8]相应结果的改进和推广. 显然，本文的讨论不存在文献[9]中所指出的情况.

1　预备知识和引理

本文论证需要以下概念和引理.

定义1　算子 $g:H\to H$ 称为

(i) ξ-强单调的，如果存在常数 $\xi>0$，使得

$$\langle g(x)-g(y),x-y\rangle\geqslant\xi\|x-y\|^2,\forall x,y\in H;$$

(ii) ζ-Lipschitz连续的，如果存在常数 $\zeta>0$，使得

$$\|g(x)-g(y)\|\leqslant\zeta\|x-y\|,\forall x,y\in H.$$

定义2　算子 $\eta:H\times H\to H$ 称为

(i) σ-强单调的，如果存在常数 $\sigma>0$，使得

$$\langle x-y,\eta(x,y)\rangle\geqslant\sigma\|x-y\|^2,\forall x,y\in H;$$

(ii) τ-Lipschitz连续的，如果存在常数 $\tau>0$，使得

$$\|\eta(x,y)\|\leqslant\tau\|x-y\|,\forall x,y\in H.$$

定义3[10]　设 $A:H\to CB(H)$ 是集值映射，$T:\underbrace{H\times H\times\cdots\times H}_{N个}\to H$，称为

(i) 依第 i 变元是 α-(A,g)-强单调的，如果存在常数 $\alpha>0$，使得

$$\langle T(\cdots,u_i,\cdots)-T(\cdots,v_i,\cdots),g(x)-g(y)\rangle\geqslant\alpha\|x-y\|^2,$$
$$\forall x,y\in H,u_i\in Ax,v_i\in Ay;$$

(ii) $(s_1,s_2,\cdots,s_N)$-Lipschitz连续的，如果存在常数 $s_1,s_2,\cdots,s_N>0$，使得

$$\|T(x_1,x_2,\cdots,x_N)-T(y_1,y_2,\cdots,y_N)\|\leqslant$$
$$s_1\|x_1-y_1\|+s_2\|x_2-y_2\|+\cdots+s_N\|x_N-y_N\|,$$
$$\forall x_i,y_i\in H,i=1,2,\cdots,N;$$

(iii) 集值映射 A 是 δ-H-Lipschitz 连续的，如果存在常数 $\delta>0$，使得

$$H(Ax,Ay)\leqslant\delta\|x-y\|,\forall x,y\in H,$$

其中 $H(\cdot,\cdot)$ 是 $CB(H)$ 的Hausdorff距离.

定义4　泛函 $f:H\times H\to\mathbf{R}\cup\{+\infty\}$ 称为关于 x 是0-对角拟凹的(简记0-$DQCV$)，如果对任意有限集 $\{x_1,\cdots,x_N\}\subset H$ 及 $y=\sum_{i=1}^n\lambda_ix_i$,$\lambda_i\geqslant0$, $\sum_{i=1}^n\lambda_i=1$，有

$$\min_{1\leqslant i\leqslant n}f(x_i,y)\leqslant0.$$

定义5　设 $\eta:H\times H\to H$ 是单值映射，$\varphi:H\to\mathbf{R}\cup\{+\infty\}$ 是真泛函，$x\in H$.如果 $\exists x^*\in H$，使得

$$\langle x^*,\eta(y,x)\rangle\leqslant\varphi(y)-\varphi(x),\forall y\in H,$$

则称 x^* 是 φ 在 x 处的 η-次梯度. φ 在 x 处的 η-次梯度的全体记为 $\partial_\eta\varphi(x)$，即

$$\partial_\eta\varphi(x)=\left\{x^*\,\middle|\,\langle x^*,\eta(y,x)\rangle\leqslant\varphi(y)-\varphi(x),\forall y\in H\right\}.\quad(4)$$

如果 $\partial_\eta\varphi(x)\neq\varnothing$，则称 φ 在 x 处是 η-次可微的，并称 $\partial_\eta\varphi(x)$ 是 φ 在 x 处的 η-次微分.

定义6　设 η,φ 如定义5，如果对每个 $x\in H$，存在唯一 $u\in H$ 使得

$$\langle u-x,\eta(y,u)\rangle\geqslant\rho\varphi(u)-\rho\varphi(y),\forall y\in H,$$

则映射 $x\mapsto u$ 称为 φ 的 η-近似映射，也叫豫解算子，记为 J_φ^ρ，其中 ρ 为正数. 由(4)式及 J_φ^ρ 定义，有 $x-u\in\rho\partial_\eta\varphi(u)$，从而

$$J_\varphi^\rho(x)=(I+\rho\partial_\eta\varphi)^{-1}(x).$$

引理1　设 $\eta:H\times H\to H$ 是连续 σ-强单调算子，$\eta(x,y)=-\eta(y,x),\ x,y\in H$. 对 $x\in H,h(y,u)=\langle x-u,\eta(y,u)\rangle$ 关于 y 是0-$DQCV$，又设 $\varphi:H\to\mathbf{R}\cup\{+\infty\}$ 是下半连续 η-次可微真泛函，则对任意给出的 $\rho>0,x\in H$，存在唯一的 $u\in H$，使得

$$\langle u-x,\eta(y,u)\rangle\geqslant\rho\varphi(u)-\rho\varphi(y),\forall y\in H,$$

即有 $u=J_\varphi^\rho(x)$.

引理2　设 $\eta:H\times H\to H$ 是 σ-强单调和 τ-Lipschitz 连续映射[11]，$\eta(x,y)=-\eta(y,x)$，设 $h(y,u)$, φ,ρ 如引理1所叙，则 φ 的 η-近似 $J_\varphi^\rho(x)$ 是 τ/σ-Lipschitz 连续的.

2　主要结果

先给出下面的定理.

定理1　$(x_1^*,x_2^*,\cdots,x_N^*;\ u_1^*,\ u_2^*,\cdots,\ u_N^*)$ 是问题(1)的解当且仅当 $(x_1^*,x_2^*,\cdots,x_N^*;\ u_1^*,\ u_2^*,\cdots,\ u_N^*)$ 满足以下等式:

$$\begin{cases}g_1(x_1^*)=J_{\varphi_1}^{\rho_1}(g_1(x_1^*)-\rho_1T_1(u_1^*,\ u_2^*,\cdots,\ u_N^*)),\\ g_2(x_2^*)=J_{\varphi_2}^{\rho_2}(g_2(x_2^*)-\rho_2T_2(u_1^*,\ u_2^*,\cdots,\ u_N^*)),\\ \qquad\vdots\\ g_N(x_N^*)=J_{\varphi_N}^{\rho_N}(g_N(x_N^*)-\rho_NT_N(u_1^*,\ u_2^*,\cdots,\ u_N^*)),\end{cases}\quad(5)$$

其中 $J_{\varphi_i}^{\rho_i}=(I+\rho_i\partial_\eta\varphi_i)^{-1},\partial_{\eta_i}\varphi_i(x)\neq\varnothing,\forall x\in H,i=1,2,\cdots,N.$

证　设(5)式成立，由 $J_{\varphi_i}^{\rho_i}=(I+\rho_i\partial_{\eta_i}\phi_i)^{-1}$ 有

$$g_i(x_i^*)+\rho_i\partial_{\eta_i}\varphi_i(g_i(x_i^*))\in g_i(x_i^*)-\rho_iT_i(u_1^*,\ u_2^*,\cdots,\ u_N^*),$$

即

$$-T_i(u_1^*, u_2^*, \cdots, u_N^*) \in \partial_{\eta_i}\varphi_i(g_i(x_i^*)).$$

由 η_i-次微分定义，上式成立当且仅当

$$-\langle T_i(u_1^*, u_2^*, \cdots, u_N^*),\ \eta_i(x, g_i(x_i^*))\rangle \leqslant \varphi_i(x) - \varphi_i(g_i(x_i^*)),\ \forall x \in H,$$

即

$$\langle T_i(u_1^*, u_2^*, \cdots, u_N^*),\ \eta_i(x, g_i(x_i^*))\rangle \geqslant \varphi_i(g_i(x_i^*)) - \varphi_i(x),$$
$$\forall x \in H,\ i = 1, 2, \cdots, N.$$

故 $(x_1^*, x_2^*, \cdots, x_N^*;\ u_1^*, u_2^*, \cdots, u_N^*)$ 是问题(1)的解.

下面给出问题(1)的解的迭代算法.

算法(I) 任取 $x_1^0, x_2^0, \cdots, x_N^0 \in H$, $u_1^0 \in A_1x_1^0, u_2^0 \in A_2x_2^0, \cdots, u_N^0 \in A_Nx_N^0$，设

$$x_1^1 = x_1^0 - g_1(x_1^0) + J_{\varphi_1}^{\rho_1}(g_1(x_1^0) - \rho_1T_1(u_1^0, u_2^0, \cdots, u_N^0)),$$
$$x_2^1 = x_2^0 - g_2(x_2^0) + J_{\varphi_2}^{\rho_2}(g_2(x_2^0) - \rho_2T_2(u_1^0, u_2^0, \cdots, u_N^0)),$$
$$\vdots$$
$$x_N^1 = x_N^0 - g_N(x_N^0) + J_{\varphi_N}^{\rho_N}(g_N(x_N^0) - \rho_NT_N(u_1^0, u_2^0, \cdots, u_N^0)).$$

由文献[12]知，对 $i = 1, 2, \cdots, N$，$\exists u_i^1 \in A_ix_i^1$，使

$$\|u_i^0 - u_i^1\| \leqslant (1+1)H(A_ix_i^0, A_ix_i^1), i = 1, 2, \cdots, N,$$

有

$$x_i^2 = x_i^1 - g_i(x_i^1) + J_{\varphi_i}^{\rho_i}(g_i(x_i^1) - \rho_iT_i(u_1^1, u_2^1, \cdots, u_N^1)), i = 1, 2, \cdots, N.$$

由归纳可以定义 $\{x_i^n\}, \{u_i^n\}$，它们满足

$$x_i^{n+1} = x_i^n - g_i(x_i^n) + J_{\varphi_i}^{\rho_i}(g_i(x_i^n) - \rho_iT_i(u_1^n, u_2^n, \cdots, u_N^n)),$$

其中 $u_i^n \in A_i^nx_i^n$，$\|u_i^n - u_i^{n+1}\| \leqslant [1 + 1/(n+1)]H(A_i(x_i^n), A_i(x_i^{n+1})), i = 1, 2, \cdots, N, n = 0, 1, 2, \cdots$.

定理2 设 H 为实Hilbert空间，对 $i = 1, 2, \cdots, N$，设 $A_i: H \to CB(H)$ 是 δ_i-H-Lipschitz 连续的集值映射，$\eta_i: H \times H \to H$ 是 σ_i-强单调和 τ_i-Lipschitz 连续映射，满足 $\eta_i(x, y) = -\eta_i(y, x)$，$g_i: H \to H$ 是 ξ_i-强单调和 ζ_i-Lipschitz 连续映射，$T_i: \underbrace{H \times H \times \cdots \times H}_{N个} \to H$ 是 $(s_1, \cdots, s_N)$-Lipschitz 连续映射，它关于第 i 变元还是 α_i-(A_i, g_i)-强单调的.设 $\varphi_i: H \to \mathbf{R} \cup \{+\infty\}$ 是下半连续 η_i-次可微的真泛函，假设 $h_i(y, u) = \langle x - g_i(u), y_i(y, u)\rangle$ 关于 y 是0-$DQCV$，如果 $\exists \rho_1, \cdots, \rho_N > 0$，使得

$$(1 - 2\xi_1 + \zeta_1^2)^{1/2} + \frac{\tau_1}{\sigma_1}(\xi_1^2 - 2\rho_1\alpha_1 + \rho_1^2s_1^2\delta_1^2)^{1/2} +$$
$$\frac{\tau_2}{\sigma_2}\rho_2s_1\delta_1 + \cdots + \frac{\tau_N}{\sigma_N}\rho_Ns_1\delta_1 < 1,$$
$$(1 - 2\xi_2 + \zeta_2^2)^{1/2} + \frac{\tau_2}{\sigma_2}(\xi_2^2 - 2\rho_2\alpha_2 + \rho_2^2s_2^2\delta_2^2)^{1/2} +$$
$$\frac{\tau_1}{\sigma_1}\rho_1s_2\delta_2 + \frac{\tau_3}{\sigma_3}\rho_3s_2\delta_2 + \cdots + \frac{\tau_N}{\sigma_N}\rho_Ns_2\delta_2 < 1,$$
$$\vdots$$
$$(1 - 2\xi_N + \zeta_N^2)^{1/2} + \frac{\tau_N}{\sigma_N}(\xi_N^2 - 2\rho_N\alpha_N + \rho_N^2s_N^2\delta_N^2)^{1/2} +$$
$$\frac{\tau_1}{\sigma_1}\rho_1s_N\delta_N + \cdots + \frac{\tau_{N-1}}{\sigma_{N-1}}\rho_{N-1}s_N\delta_N < 1, \tag{6}$$

则由迭代算法(I)得到的序列 $\{x_1^n\}, \{x_2^n\}, \cdots, \{x_N^n\}$, $\{u_1^n\}, \{u_2^n\}, \cdots, \{u_N^n\}$，分别强收敛于问题(1)的解 $x_1^*, x_2^*, \cdots, x_N^*, u_1^*, u_2^*, \cdots, u_N^*$.

证 由算法(I)得

$$\|x_1^{n+1} - x_1^n\| = \|x_1^n - g_1(x_1^n) + J_{\varphi_1}^{\rho_1}(g_1(x_1^n) - \rho_1T_1(u_1^n, u_2^n, \cdots,$$
$$u_N^n)) - x_1^{n-1} + g_1(x_1^{n-1}) - J_{\varphi_1}^{\rho_1}(g_1(x_1^{n-1}) -$$
$$\rho_1T_1(u_1^{n-1}, u_2^{n-1}, \cdots, u_N^{n-1}))\| \leqslant$$
$$\|x_1^n - x_1^{n-1} - g_1(x_1^n) + g_1(x_1^{n-1})\| + \frac{\tau_1}{\sigma_1}\|g_1(x_1^n) - g_1(x_1^{n-1}) -$$
$$\rho_1T_1(u_1^n, \cdots, u_N^n) + \rho_1T_1(u_1^{n-1}, \cdots, u_N^{n-1})\|. \tag{7}$$

由 g_1 是 ξ_1-强单调和 ζ_1-Lipschitz 连续的，故有

$$\|x_1^n - x_1^{n-1} - (g_1(x_1^n) - g_1(x_1^{n-1}))\| \leqslant \sqrt{1 - 2\xi_1 + \zeta_1^2}\|x_1^n - x_1^{n-1}\|. \tag{8}$$

注意到，

$$\|g_1(x_1^n) - g_1(x_1^{n-1}) - \rho_1(T_1(u_1^n, \cdots, u_N^n) -$$
$$T_1(u_1^{n-1}, \cdots, u_N^{n-1}))\| \leqslant \|g_1(x_1^n) - g_1(x_1^{n-1}) -$$
$$\rho_1(T_1(u_1^n, u_2^n, \cdots, u_N^n) - T_1(u_1^{n-1}, u_2^n, \cdots, u_N^n))\| +$$
$$\rho_1\|T_1(u_1^{n-1}, u_2^n, \cdots, u_N^n) - T_1(u_1^{n-1}, u_2^{n-1}, \cdots, u_N^{n-1})\|, \tag{9}$$

而 T_1 是 $(s_1, \cdots, s_N)$-Lipschitz 连续和关于第1变元是 α_1-(A_1, g_1)-强单调的，故有

$$\|g_1(x_1^n) - g_1(x_1^{n-1}) - \rho_1(T_1(u_1^n, u_2^n, \cdots, u_N^n) -$$
$$T_1(u_1^{n-1}, u_2^n, \cdots, u_N^n))\| \leqslant [\xi_1^2 - 2\rho_1\alpha_1 + \rho_1^2s_1^2\delta_1^2(1 +$$
$$1/n)^2]^{1/2}\|x_1^n - x_1^{n-1}\| \tag{10}$$

以及

$$\rho_1\|T_1(u_1^{n-1}, u_2^n, \cdots, u_N^n) - T_1(u_1^{n-1}, u_2^{n-1}, \cdots, u_N^{n-1})\| \leqslant$$
$$\rho_1(1 + 1/n)[s_2\delta_2\|x_2^n - x_2^{n-1}\| + s_3\delta_3\|x_3^n - x_3^{n-1}\| +$$
$$\cdots + s_N\delta_N\|x_N^n - x_N^{n-1}\|]. \tag{11}$$

于是由(6)~(11)式，有

$$\|x_1^{n+1} - x_1^n\| \leqslant \left[\sqrt{1 - 2\xi_1 + \zeta_1^2} + \frac{\tau_1}{\sigma_1}\left(\xi_1^2 -\right.\right.$$
$$\left.\left.2\rho_1\alpha_1 + \rho_1^2s_1^2\delta_1^2(1 + 1/n)^2\right)^{1/2}\right]\|x_1^n - x_1^{n-1}\| +$$
$$\rho_1\frac{\tau_1}{\sigma_1}(1 + 1/n)[s_2\delta_2\|x_2^n - x_2^{n-1}\| +$$
$$s_3\delta_3\|x_3^n - x_3^{n-1}\| + \cdots + s_N\delta_N\|x_N^n - x_N^{n-1}\|].$$

一般地，对 $i = 1, 2, \cdots, N$，

$$\left\|x_i^{n+1}-x_i^n\right\| \leqslant \left[\sqrt{1-2\xi_i+\zeta_i^2}+\frac{\tau_i}{\sigma_i}\left(\xi_i^2-2\rho_i\alpha_i+\rho_i^2 s_i^2\delta_i^2(1+1/n)^2\right)^{1/2}\right]\left\|x_i^n-x_i^{n-1}\right\|+\rho_i\frac{\tau_i}{\sigma_i}(1+1/n)\left[s_1\delta_1\left\|x_1^n-x_1^{n-1}\right\|+s_2\delta_2\left\|x_2^n-x_2^{n-1}\right\|+\cdots+s_{i-1}\delta_{i-1}\left\|x_{i-1}^n-x_{i-1}^{n-1}\right\|+s_{i+1}\delta_{i+1}\left\|x_{i+1}^n-x_{i+1}^{n-1}\right\|+\cdots+s_N\delta_N\left\|x_N^n-x_N^{n-1}\right\|\right].$$

于是

$$\left\|x_1^{n+1}-x_1^n\right\|+\left\|x_2^{n+1}-x_2^n\right\|+\cdots+\left\|x_N^{n+1}-x_N^n\right\| \leqslant \theta_n\left(\left\|x_1^n-x_1^{n-1}\right\|+\left\|x_2^n-x_2^{n-1}\right\|+\cdots+\left\|x_N^n-x_N^{n-1}\right\|\right).$$

其中

$$\theta_n=\max\left\{\sqrt{1-2\xi_1+\zeta_1^2}+\frac{\tau_1}{\sigma_1}\left(\xi_1^2-2\rho_1\alpha_1+\rho_1^2 s_1^2\delta_1^2(1+\frac{1}{n})^2\right)^{1/2}+\rho_2\frac{\tau_2}{\sigma_2}s_1\delta_1(1+\frac{1}{n})+\cdots+\rho_N\frac{\tau_N}{\sigma_N}s_1\delta_1(1+\frac{1}{n}),\sqrt{1-2\xi_2+\zeta_2^2}+\frac{\tau_2}{\sigma_2}\left(\xi_2^2-2\rho_2\alpha_2+\rho_2^2 s_2^2\delta_2^2(1+\frac{1}{n})^2\right)^{1/2}+\rho_1\frac{\tau_1}{\sigma_1}s_2\delta_2(1+\frac{1}{n})+\rho_3\frac{\tau_3}{\sigma_3}s_2\delta_2(1+\frac{1}{n})+\cdots+\rho_N\frac{\tau_N}{\sigma_N}s_2\delta_2(1+\frac{1}{n}),\cdots\sqrt{1-2\xi_N+\zeta_N^2}+\frac{\tau_N}{\sigma_N}\left(\xi_N^2-2\rho_N\alpha_N+\rho_N^2 s_N^2\delta_N^2(1+\frac{1}{n})^2\right)^{1/2}+\rho_1\frac{\tau_1}{\sigma_1}s_N\delta_N(1+\frac{1}{n})+\rho_2\frac{\tau_2}{\sigma_2}s_N\delta_N(1+\frac{1}{n})+\cdots+\rho_{N-1}\frac{\tau_{N-1}}{\sigma_{N-1}}s_N\delta_N(1+\frac{1}{n})\right\}.$$

记

$$\theta=\max\left\{\sqrt{1-2\xi_1+\zeta_1^2}+\frac{\tau_1}{\sigma_1}\left(\xi_1^2-2\rho_1\alpha_1+\rho_1^2 s_1^2\delta_1^2\right)^{1/2}+\rho_2\frac{\tau_2}{\sigma_2}s_1\delta_1+\cdots+\rho_N\frac{\tau_N}{\sigma_N}s_1\delta_1,\sqrt{1-2\xi_2+\zeta_2^2}+\frac{\tau_2}{\sigma_2}\left(\xi_2^2-2\rho_2\alpha_2+\rho_2^2 s_2^2\delta_2^2\right)^{1/2}+\rho_1\frac{\tau_1}{\sigma_1}s_2\delta_2+\rho_3\frac{\tau_3}{\sigma_3}s_2\delta_2+\cdots+\rho_N\frac{\tau_N}{\sigma_N}s_2\delta_2,\cdots\sqrt{1-2\xi_N+\zeta_N^2}+\frac{\tau_N}{\sigma_N}\left(\xi_N^2-2\rho_N\alpha_N+\rho_N^2 s_N^2\delta_N^2\right)^{1/2}+\rho_1\frac{\tau_1}{\sigma_1}s_N\delta_N+\rho_2\frac{\tau_2}{\sigma_2}s_N\delta_N+\cdots+\rho_{N-1}\frac{\tau_{N-1}}{\sigma_{N-1}}s_N\delta_N\right\},$$

则由(6)式可知 $0<\theta<1$，从而可知 $\{x_1^n\},\cdots,\{x_N^n\}$ 均为Cauchy列，于是$\exists x_1^*,\cdots,x_N^*\in H$，使 $x_i^n\to x_i^*(n\to\infty)$，$i=1,2,\cdots,N$.下证 $u_i^n\to u_i^*, i=1,2,\cdots,N$.由

$$\left\|u_i^n-u_i^{n-1}\right\| \leqslant \left(1+\frac{1}{n}\right)H\left(A_ix_i^n,A_ix_i^{n-1}\right) \leqslant \left(1+\frac{1}{n}\right)\delta_i\left\|x_i^n-x_i^{n-1}\right\|$$

可知 $\{u_i^n\}$ 也是Cauchy序列，故 $\exists u_i^*\in H$，使得 $u_i^n\to u_i^*(n\to\infty)$.

又由

$$d(u_i^*,A_ix_i^*) \leqslant \left\|u_i^*-u_i^n\right\|+d(u_i^n,A_ix_i^*) \leqslant \left\|u_i^*-u_i^n\right\|+H\left(A_ix_i^n,A_ix_i^*\right) \leqslant \left\|u_i^*-u_i^n\right\|+\delta_i\left\|x_i^n-x_i^*\right\|\to 0(n\to\infty).$$

所以，$d(u_i^*,A_ix_i^*)=0$，由 $A_ix_i^*$ 是闭的，故有 $u_i^*\in A_ix_i^*, i=1,2,\cdots,N$.由

$$x_i^{n+1}=x_i^n-g_i(x_i^n)+J_{\varphi_i}^{\rho_i}(g_i(x_i^n)-\rho_iT_i(u_1^n, u_2^n,\cdots, u_N^n)(i=1,2,\cdots,N)$$

及 g_i, J_{φ_i}, T_i 的连续性，令 $n\to\infty$ 就有

$$0=-g_i(x_i^*)+J_{\varphi_i}^{\rho_i}(g_i(x_i^*)-\rho_iT_i(u_1^*, u_2^*,\cdots, u_N^*))$$

及

$$u_1^*\in A_1x_1^*, u_2^*\in A_2x_2^*,\cdots,u_N^*\in A_Nx_N^*,$$

由定理1知，定理2得证.

注1　T_i, A_i, η_i, g_i和φ_i 的不同选取，定理2可以归结为许多熟知的广义拟-似变分包含组(不等式组)的结果．这里 φ_i 非凸，A_i 非紧值.

3　参考文献

[1] Ding Xieping. Generalized quasi-variational-like inclusions with nonconvex functional [J]. Appl Math Comput, 2001, 122: 267-282.

[2] He Zhenhua , Gu Feng. Generalized system for relaxed cocoercive mixed variational inequalities in Hilbert spaces [J]. Appl Math Comput, 2009, 214(1): 26-30.

[3] Chang S S , Joseph Lee H W , Chan C K. Generalized system for relaxed cocoercive variational inequalities in Hilbert spaces [J]. Appl Math Letters, 2007, 20(3): 329-334.

[4] Verma R U. General convergence analysis for two-step projection methods and applications to variational problems [J]. Appl Math Letters, 2005, 18(11): 1286-1292.

[5] Verma R U. Projection methods, algorithms and a new system of nonlinear variational inequalities [J].Comput Math Appl, 2001, 42(7/8): 1025-1031.

[6] Huang Zhenyu , Noor M A. An explicit projection method for a system of nonlinear variational inequalities with different (γ, r)- cocoercive mapping [J]. Appl Math Comput, 2007(1), 190: 356-361.

[7] Hajjafar A, Verma R U. General approximation solvability of a system of strongly *g*-*r*-pseudomonotonic nonlinear variational inequalities and projection methods [J]. Math Comput Modelling, 2006, 43(1/2): 150-157.

[8] Ahmad R , Siddiqi A H, Khan Z. Proximal point algorithm for generalized multivalued nonlinear quasi-variational-like inclusions in Banach Spaces [J]. Appl Math Comput, 2005, 163(1): 295-308.

[9] Yang Qingzhi. On a generalized system for relaxed cocoercive variational inequalities and projection methods [J]. J Optim Theory Appl, 2006, 130(3): 545-547.

[10] Peng Jianwen, Zhu Daoli. A new system of generalized mixed quasi-variational inclusion with (H, η)-monotone operators [J]. J Math Anal Appl, 2007, 327(1): 157-187.

[11] 张旭, 叶志强, Banach 空间中-类含 H-增生算子的新型广义非线性混合似变分包含组[J]. 重庆师范大学学报: 自然科学版, 2006,23(4): 6-9, 24.

[12] Nadler S B. Multi-valued contraction mappings [J].Pacific J Math, 1969, 30(2): 475-486.

A System of Generalized Quasi-Variational-Like Inclusions with η-Subdifferentiable Mappings

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Abstract: The existence of solutions for the system of generalized quasi-variational-like inclusions with η-subdifferentiable mappings and the convergence of some N-step iterative algorithms are proved by using the η-proximal mapping technique, which extend and improve some known results in the literature.

Key words: system of generalized quasi-variational-like inclusions; η-proximal mapping; monotone mapping; iterative algorithm

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