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一类反周期函数的双周期缺项插值问题

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摘要: 讨论了以 π 为周期的反周期函数双周期插值问题,建立了关于该插值问题基多项式的方程组,利用克莱默法则给出插值问题有解的充要条件,并给出该条件下插值解的表达式.

关键词: 反周期函数; 双周期; 插值

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0 引言

文献[1-6]研究了多种类型的三角插值问题与整插值问题,文献[4]中给出关于 π-周期及其反周期三角多项式空间与基多项式的概念,同时定义了 Sharma-Varma 算子,本文所用记号与文献[4]完全一致,在此不做叙述. 文献[7]给出反周期函数的双周期$(0,m_1,m_2,m_3)$插值问题,即 $\forall\ \alpha_{0k},\alpha_{1k},\alpha_{2k},\alpha_{3k}(k=0,1,\cdots,n-1)$,是否存在 π-反周期的三角多项式 $T(x)$,满足条件:

$$T(x_{2k})=\alpha_{0,k},T^{(m_1)}(x_{2k+1})=\alpha_{1,k},$$
$$T^{(m_2)}(x_{2k+1})=\alpha_{2,k},T^{(m_3)}(x_{2k+1})=\alpha_{3,k}.$$

本文在文献[8]的研究基础上通过改变结点组的位置考虑类似的双周期问题,即 $\forall\ \alpha_{0k},\alpha_{1k},\alpha_{2k},\alpha_{3k}(k=1,2,\cdots,n)$,是否存在 π-反周期的三角多项式 $T(x)$,满足条件:

$$T(x_{2k+1})=\alpha_{0,k},T^{(m_1)}(x_{2k})=\alpha_{1,k},$$
$$T^{(m_2)}(x_{2k})=\alpha_{2,k},T^{(m_3)}(x_{2k})=\alpha_{3,k},\tag{1}$$

其中 $m_1<m_2<m_3$ 为正整数; $x_{2k}=2k\pi/(2n)$, $x_{2k+1}=(2k+1)\pi/(2n)$; $\{\alpha_{j,k}\}$ 为给定复数列, $k=0,1,\cdots,n-1$, $j=0,1,2,3$.

1 主要引理

为了证明结论,给出引理 1.

引理 1 若 n 为偶数, $T(x)\in\omega_{4n-1}^{\perp}$,则 $\exists\ U_j\in\omega_{n-1}^{\perp}$, $j=0,1,2,3$,使得

$$T(x)=\frac{1}{2}\left[\cos 3nx\left(U_0-\frac{1}{\sqrt{2}}U_1+\frac{1}{\sqrt{2}}U_3\right)+\sin 3nx\left(\frac{1}{\sqrt{2}}U_1-U_2+\frac{1}{\sqrt{2}}U_3\right)\right]+\frac{1}{2}\left[\cos nx\left(U_0+\frac{1}{\sqrt{2}}U_1-\frac{1}{\sqrt{2}}U_3\right)+\sin nx\left(\frac{1}{\sqrt{2}}U_1+U_2+\frac{1}{\sqrt{2}}U_3\right)\right],\tag{2}$$

其中 $U_j=U_j(x)=\sum\limits_{k=0}^{n-1}(-1)^kT(z_{4k+j})L_n(x-z_{4k+j})$, $j=0,1,2,3$.

若 n 为奇数,则 $\exists\ U_j\in\omega_{n-1}$, $j=0,1,2,3$,使得(2)式仍然成立. 为了符号简洁,记

$$V_1=\frac{1}{2}\left(U_0-\frac{1}{\sqrt{2}}U_1+\frac{1}{\sqrt{2}}U_3\right),$$
$$V_2=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_1-U_2+\frac{1}{\sqrt{2}}U_3\right),$$
$$V_3=\frac{1}{2}\left(U_0+\frac{1}{\sqrt{2}}U_1-\frac{1}{\sqrt{2}}U_3\right),$$
$$V_4=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_1+U_2+\frac{1}{\sqrt{2}}U_3\right),\tag{3}$$

则 $\forall\ T(x)\in\omega_{4n-1}^{\perp}$,有

$$T(x)=\cos 3nxV_1+\sin 3nxV_2+\cos nxV_3+\sin nxV_4.\tag{4}$$

且从(3)式可知,

$$V_1+V_3=U_0=\sum_{k=0}^{n-1}T(-k)^k(z_{4k})L_n(x-z_{4k})=\sum_{k=0}^{n-1}(-k)^kT(x_{2k})L_n(x-x_{2k}),$$
$$V_4-V_2=U_2=\sum_{k=0}^{n-1}(-1)^kT(z_{4k+2})L_n(x-z_{4k+2})=\sum_{k=0}^{n-1}(-1)^kT(x_{2k+1})L_n(x-x_{2k+1}).\tag{5}$$

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利用(4)式结合文献[4]中的结果及算子的线性不难得到引理2.

引理2　若$T(x)\in\omega_{4n-1}^{\perp}$,有
$D^mT(x)=\cos 3nx(A_{3n}^mV_1+B_{3n}^mV_2)+\sin 3nx(A_{3n}^mV_2-B_{3n}^mV_1)+\cos nx(A_n^mV_3+B_n^mV_4)+\sin nx(A_n^mV_4-B_n^mV_3)$,其中$m$为正整数;$D^m$表示$m$阶微分算子.

2　主要结论

定理1　若$m_1<m_2<m_3$为正整数,则$\exists T(x)\in\omega_{4n-1}^{\perp}$满足(1)式当且仅当$m_1,m_2,m_3$中有2个偶数1个奇数,且

$$T(x)=\sum_{k=0}^{n-1}[\alpha_{0k}T_0(x-x_{2k+1})\sum_{u=1}^{3}\alpha_{uk}T_u(x-x_{2k})],$$

其中$T_j(x)=\cos 3nxV_{j1}+\sin 3nxV_{j2}+\cos nxV_{j3}+\sin nxV_{j4}$,$V_{j1}=\frac{1}{2}\left(U_{j0}-\frac{1}{\sqrt{2}}U_{j1}+\frac{1}{\sqrt{2}}U_{j3}\right)$,$V_{j2}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{j1}-U_{j2}+\frac{1}{\sqrt{2}}U_{j3}\right)$,$V_{j3}=\frac{1}{2}\left(U_{j0}+\frac{1}{\sqrt{2}}U_{j1}-\frac{1}{\sqrt{2}}U_{j3}\right)$,$V_{j4}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{j1}+U_{j2}+\frac{1}{\sqrt{2}}U_{j3}\right)$,$U_{ji}=\frac{D_{ji}}{D}$,

$$D=\frac{1}{16}\begin{vmatrix}0&0&1&0\\a_n^{m_1}+a_{3n}^{m_1}&a_n^{m_1}-a_{3n}^{m_1}+b_n^{m_1}+b_{3n}^{m_1}&b_n^{m_1}-b_{3n}^{m_1}&a_{3n}^{m_1}-a_n^{m_1}+b_n^{m_1}+b_{3n}^{m_1}\\a_n^{m_2}+a_{3n}^{m_2}&a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2}&b_n^{m_2}-b_{3n}^{m_2}&a_{3n}^{m_2}-a_n^{m_2}+b_n^{m_2}+b_{3n}^{m_2}\\a_n^{m_3}+a_{3n}^{m_3}&a_n^{m_3}-a_{3n}^{m_3}+b_n^{m_3}+b_{3n}^{m_3}&b_n^{m_3}-b_{3n}^{m_3}&a_{3n}^{m_3}-a_n^{m_3}+b_n^{m_3}+b_{3n}^{m_3}\end{vmatrix},$$

D_{ji}是分别用$(L_n(x-x_2),0,0,0)^{\mathrm{T}}$,$(0,L_n(x),0,0)^{\mathrm{T}}$,$(0,0,L_n(x),0)^{\mathrm{T}}$,$(0,0,0,L_n(x))^{\mathrm{T}}$代替$D$的第$i+1$列产生的行列式,$i=0,1,2,3$,$j=0,1,2,3$.

3　定理1的证明

要$\exists T(x)\in\omega_{4n-1}^{\perp}$满足(1)式,需有基函数$T_0(x),T_j(x)\in\omega_{4n-1}^{\perp}$,使得$T_0(x_{2k+1})=\delta_{0k}$,$T_0^{(m_j)}(x_{2k})=0$,$T_j(x_{2k+1})=0$,$T_j^{(m_j)}(x_{2k})=\delta_{0k}$,$T_j^{(m_v)}(x_{2k})=0$,$j,v=1,2,3$,$j\neq v$,$k=0,\cdots,n-1$.

先看$T_0(x)$:$T_0(x_{2k+1})=\delta_{0k}$,$T_0^{(m_1)}(x_{2k})=0$,$T_0^{(m_2)}(x_{2k})=0$,$T_0^{(m_3)}(x_{2k})=0$.

由引理1可设$T_0(x)=\cos 3nxV_{01}+\sin 3nxV_{02}+\cos nxV_{03}+\sin nxV_{04}$,其中

$$V_{01}=\frac{1}{2}\left(U_{00}-\frac{1}{\sqrt{2}}U_{01}+\frac{1}{\sqrt{2}}U_{03}\right),$$

$$V_{02}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{01}-U_{02}+\frac{1}{\sqrt{2}}U_{03}\right),$$

$$V_{03}=\frac{1}{2}\left(U_{00}+\frac{1}{\sqrt{2}}U_{01}-\frac{1}{\sqrt{2}}U_{03}\right),$$

$$V_{04}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{01}+U_{02}+\frac{1}{\sqrt{2}}U_{03}\right),$$

$$U_{02}=\sum_{k=0}^{n-1}(-1)^kT_0(z_{4k+2})L_n(x-z_{4k+2})=\sum_{k=0}^{n-1}(-1)^kT_0(x_{2k+1})L_n(x-x_{2k+1})=L_n(x-x_2).\quad(6)$$

再利用引理2,有

$T_0^{(m_j)}(x_{2k})=\cos 3nx_{2k}(A_{3n}^{m_j}V_{01}+B_{3n}^{m_j}V_{02})+\sin 3nx_{2k}\cdot(A_{3n}^{m_j}V_{02}-B_{3n}^{m_j}V_{01})+\cos nx_{2k}(A_n^{m_j}V_{03}+B_n^{m_j}V_{04})+\sin nx_{2k}(A_n^{m_j}V_{04}-B_n^{m_j}V_{03})=0$,$j=1,2,3$,

故

$$A_{3n}^{m_j}V_{01}+B_{3n}^{m_j}V_{02}+A_n^{m_j}V_{03}+B_n^{m_j}V_{04}=0.\quad(7)$$

联立(6)式和(7)式得到方程组

$$\begin{cases}U_{02}=L_n(x-x_2),\\ \dfrac{(a_n^{m_j}+a_{3n}^{m_j})U_{00}}{2}-b_{3n}^{m_j}+\dfrac{(a_n^{m_j}-a_{3n}^{m_j}+b_n^{m_j}+b_{3n}^{m_j})U_{01}}{2\sqrt{2}}+\\ \dfrac{(b_n^{m_j}-b_{3n}^{m_j})U_{02}}{2}+\dfrac{(a_{3n}^{m_j}-a_n^{m_j}+b_n^{m_j}+b_{3n}^{m_j})U_{03}}{2\sqrt{2}}=0,\end{cases}$$

$j=1,2,3$.

再看$T_1(x)$:$T_1(x_{2k+1})=0$,$T_1^{(m_1)}(x_{2k})=\delta_{0k}$,$T_1^{(m_2)}(x_{2k})=0$,$T_1^{(m_3)}(x_{2k})=0$

可设$T_1(x)=\cos 3nxV_{11}+\sin 3nxV_{12}+\cos nxV_{13}+\sin nxV_{14}$,其中

$$V_{11}=\frac{1}{2}\left(U_{10}-\frac{1}{\sqrt{2}}U_{11}+\frac{1}{\sqrt{2}}U_{13}\right),$$

$$V_{12}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{11}-U_{12}+\frac{1}{\sqrt{2}}U_{13}\right),$$

$$V_{13}=\frac{1}{2}\left(U_{10}+\frac{1}{\sqrt{2}}U_{11}-\frac{1}{\sqrt{2}}U_{13}\right),$$

$$V_{14}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{11}+U_{12}+\frac{1}{\sqrt{2}}U_{13}\right),$$

$$U_{12}=\sum_{k=0}^{n-1}(-1)^kT_1(z_{4k+2})L_n(x-z_{4k+2})=\sum_{k=0}^{n-1}(-1)^kT_1(x_{2k+1})L_n(x-x_{2k+1})=0,\quad(8)$$

$T_1^{(m_j)}(x)=\cos 3nx(A_{3n}^{m_j}V_{11}+B_{3n}^{m_j}V_{12})+\sin 3nx(A_{3n}^{m_j}V_{12}-$

$B_{3n}^{m_j}V_{11}) + \cos nx(A_n^{m_j}V_{13} + B_n^{m_j}V_{14}) + \sin nx(A_n^{m_j}V_{14} - B_n^{m_j}V_{13})$，$j = 1,2,3$.

由(5)式可得

$$A_{3n}^{m_j}V_{11} + B_{3n}^{m_j}V_{12} + A_n^{m_j}V_{13} + B_n^{m_j}V_{14} = \sum_{k=0}^{n-1}(-1)^k T_1^{(m_j)}(x_{2k})L_n(x - x_{2k}) = \begin{cases} L_n(x), & j = 1, \\ 0, & j = 2,3. \end{cases} \tag{9}$$

联立(8)式和(9)式得到方程组

$$\begin{cases} U_{12} = 0, \\ \dfrac{(a_n^{m_1} + a_{3n}^{m_1})U_{10}}{2} + \dfrac{(a_n^{m_1} - a_{3n}^{m_1} + b_n^{m_1} + b_{3n}^{m_1})U_{11}}{2\sqrt{2}} + \dfrac{(b_n^{m_1} - b_{3n}^{m_1})U_{12}}{2} + \dfrac{(a_{3n}^{m_1} - a_n^{m_1} + b_n^{m_1} + b_{3n}^{m_1})U_{13}}{2\sqrt{2}} = L(x), \\ \dfrac{(a_n^{m_l} + a_{3n}^{m_l})U_{10}}{2} + \dfrac{(a_n^{m_l} - a_{3n}^{m_l} + b_n^{m_l} + b_{3n}^{m_l})U_{11}}{2\sqrt{2}} + \dfrac{(b_n^{m_l} - b_{3n}^{m_l})U_{12}}{2} + \dfrac{(a_{3n}^{m_l} - a_n^{m_l} + b_n^{m_l} + b_{3n}^{m_l})U_{13}}{2\sqrt{2}} = 0, \end{cases}$$

$l = 2,3$. 对 $T_2(x)$，$T_3(x)$ 经过类似地讨论，又可得 2 个方程组，且 4 个方程组均有系数行列式：

$$D = \frac{1}{16}\begin{vmatrix} 0 & 0 & 1 & 0 \\ a_n^{m_1} + a_{3n}^{m_1} & a_n^{m_1} - a_{3n}^{m_1} + b_n^{m_1} + b_{3n}^{m_1} & b_n^{m_1} - b_{3n}^{m_1} & a_{3n}^{m_1} - a_n^{m_1} + b_n^{m_1} + b_{3n}^{m_1} \\ a_n^{m_2} + a_{3n}^{m_2} & a_n^{m_2} - a_{3n}^{m_2} + b_n^{m_2} + b_{3n}^{m_2} & b_n^{m_2} - b_{3n}^{m_2} & a_{3n}^{m_2} - a_n^{m_2} + b_n^{m_2} + b_{3n}^{m_2} \\ a_n^{m_3} + a_{3n}^{m_3} & a_n^{m_3} - a_{3n}^{m_3} + b_n^{m_3} + b_{3n}^{m_3} & b_n^{m_3} - b_{3n}^{m_3} & a_{3n}^{m_3} - a_n^{m_3} + b_n^{m_3} + b_{3n}^{m_3} \end{vmatrix} = -\frac{1}{4}\begin{vmatrix} a_n^{m_1} & a_{3n}^{m_1} & b_n^{m_1} + b_{3n}^{m_1} \\ a_n^{m_2} & a_{3n}^{m_2} & b_n^{m_2} + b_{3n}^{m_2} \\ a_n^{m_3} & a_{3n}^{m_3} & b_n^{m_3} + b_{3n}^{m_3} \end{vmatrix},$$

$0 \leqslant k \leqslant n-1$，当 $k = 0$ 时，记 $D = D_0$，结合 Sharma-Varma 算子的定义及行列式的性质：

$$D_0 = \frac{-i^{m_1+m_2+m_3-1}}{32}\begin{vmatrix} n^{m_1} + (-n)^{m_1} & (3n)^{m_1} + (-3n)^{m_1} & n^{m_1} - (-n)^{m_1} + (3n)^{m_1} - (-3n)^{m_1} \\ n^{m_2} + (-n)^{m_2} & (3n)^{m_2} + (-3n)^{m_2} & n^{m_2} - (-n)^{m_2} + (3n)^{m_2} - (-3n)^{m_2} \\ n^{m_3} + (-n)^{m_3} & (3n)^{m_3} + (-3n)^{m_3} & n^{m_3} - (-n)^{m_3} + (3n)^{m_3} - (-3n)^{m_3} \end{vmatrix},$$

此时 D_0 与文献[7]中得到的行列式一致，之后的讨论同文献[8]，并使用克拉默法则得到定理 1. 为简洁起见，在此省去，不做叙述.

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The Solution of 2-Periodic Lacunary Interpolation by Antiperiodic Function

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Abstract: The 2-periodic interpolation about antiperiodic trigonometric polynomials is considered，there established series of equation on basic polynomials of interpolation，a sufficient and necessary condition was obtained and the solution of interpolation was given if it existed in the end.

Key words: antiperiodic; 2-periodic; interpolation

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