文章编号:1000-5862(2014)06-0561-04

一类非线性分数阶微分方程反周期边值问题解的存在性

谢志勇,谢显华*,马 丽
(赣南师范学院数学与计算机科学学院,江西 赣州 341000)

摘要: 利用 Schauder 不动点定理和 Hölder 不等式等方法研究了一类非线性反周期分数阶微分方程边值问题,证明了当满足一定条件时其解的存在性.

关键词: 反周期分数阶微分方程;Schauder 不动点定理;Hölder 不等式;压缩映射原理

中图分类号: O 175.7 **文献标志码:** A

0 引言

分数阶常微分方程是经典的整数阶常微分方程的推广,它是将整数阶的导数用分数阶导数来替换. 而与整数阶微分方程相比,分数阶微分方程的优势在于它能更好地模拟自然界的物理过程和动态系统过程. 近年来,分数阶微分方程已作为一个重要的研究领域,它将经典微分方程推广到任意阶的情形. 现在分数阶微分方程在电子控制、多孔介质等许多领域上都有应用. 关于分数阶微分方程解的存在唯一性问题、一些特殊分数阶微分方程解的性态问题、分数阶微分方程数值解的研究参见文献[1-6].

反周期边值问题理论广泛应用于描述波动环境中物理现象的演化. 许多学者关注于分数阶微分方程反周期边值问题解的存在性和唯一性问题. 文献[6]考虑了分数阶微分方程反周期边值问题:

$$\begin{cases} {}^CD^{\beta}({}^CD^{\alpha}+\lambda)u(t)=f(t,u(t)),t\in[0,T], \\ u(0)=-u(T),u'(0)=u'(T)=0, \end{cases} \quad (1)$$

其中${}^CD^q$表示q阶 Caputo 分数阶导数,f是给定的连续函数. 研究结果主要是基于标准不动点原理得出的. 除此之外,文献[7]研究了分数阶微分方程反周期边值问题:

$$\begin{cases} {}^CD^{\alpha}x(t)=f(t,x(t),{}^CD^{q}x(t)),t\in[0,T], \\ x(0)=-x(T),{}^CD^{p}x(0)=-{}^CD^{p}x(T), \end{cases}$$

其中${}^CD^{\alpha}$表示α阶 Caputo 分数阶导数,T是 1 个正的常数,$1<\alpha\leqslant 2,0<p,q<1,\alpha-q\geqslant 1$和$f$是 1 个给定的连续函数.

方程(1)中,如果f不仅是$t,u(t)$的函数,而且也是${}^CD^{q}u(t)$的函数,那么方程所描述的物理现象将更普遍. 一个自然的问题就是其解的存在唯一性是否能够成立呢? 本文将考虑如下分数阶微分方程反周期边值问题解的存在性和唯一性:

$$\begin{cases} {}^CD^{\beta}({}^CD^{\alpha}+\lambda)u(t)=f(t,u(t),{}^CD^{q}u(t)), \\ t\in[0,T], \\ u(0)=-u(T),u'(0)=u'(T)=0, \end{cases} \quad (2)$$

其中${}^CD^{\beta}$表示β阶 Caputo 分数阶导数,T是 1 个正的常数,$1<\alpha\leqslant 2,0<\beta\leqslant 1,0<q<1,\alpha-q\geqslant 1$,$f$是 1 个给定的连续函数和$\lambda$是实数.

1 预备知识

本节将给出一些用到的基本概念、记号、定义和已有结果.

定义 1 函数$f:[0,\infty)\to\mathbf{R}$的$\alpha$阶 Caputo 分数阶导数定义为

$$^{C}D^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-s)^{n-\alpha-1}f^{(n)}(s)\,\mathrm{d}s,$$

$$n-1<\alpha<n,n=[\alpha]+1,$$

其中$[\alpha]$表示α的整数部分.

定义 2 函数$f(t),t>0$的 Riemann-Liouville α阶($\alpha>0$)分数阶积分定义为

收稿日期:2014-10-15
基金项目:国家自然科学基金(11361004),江西省教育科学“十二五”规划课题(12ZYZYB013)和江西省教学改革研究(JXJG-14-14-4)资助项目.
通信作者:谢显华(1978-),男,江西兴国人,副教授,主要从事泛函分析和微分几何的研究.

$$I^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}f(s)\,\mathrm{d}s,$$

假定右端是在$(0,\infty)$上逐点定义的.

定义 3 函数$f(t)$,$t>0$的 Riemann-Liouville $\alpha(\alpha>0)$阶分数次导数定义为

$$D^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^n\int_0^t(t-s)^{n-\alpha-1}f(s)\,\mathrm{d}s,$$

其中$n=[\alpha]+1$,假定右端是在$(0,\infty)$上逐点定义的.

引理 1 设$\alpha>0$,则分数阶微分方程${}^CD^{\alpha}u(t)=0$有形如

$$u(t)=c_0+c_1t+c_2t^2+\cdots+c_{n-1}t^{n-1}$$

的解,其中$c_i\in\mathbf{R}$,$i=0,1,2,\cdots,n-1$,$n=[\alpha]+1$.

引理 2 设$\alpha>0$,则对于某些$c_i\in\mathbf{R}$,$i=0,1,2,\cdots,n-1$,$n=[\alpha]+1$,有

$$I^{\alpha\,C}D^{\alpha}u(t)=u(t)+c_0+c_1t+c_2t^2+\cdots+c_{n-1}t^{n-1}.$$

引理 3 设E是 Banach 空间X中闭的非空凸子集,$F:E\to E$是连续映射且$F(E)$是X的相对紧致子集,则F在E中至少有 1 个不动点.

引理 4 设l和m是 2 个正数使得$1/l+1/m=1$,如果$|f(x)|^l$和$|g(x)|^m$在$[a,b]$上黎曼可积,则

$$\int_a^b|f(x)g(x)|\mathrm{d}x\leqslant\left[\int_a^b|f(x)|^l\mathrm{d}x\right]^{1/l}\cdot\left[\int_a^b|g(x)|^m\mathrm{d}x\right]^{1/m}.$$

引理 5 边值问题${}^CD^{\beta}({}^CD^{\alpha}+\lambda)u(t)=f(t,u(t),{}^CD^qu(t))$,$t\in[0,T]$,$1<\alpha\leqslant2$,$0<q<1$,$\alpha-q\geqslant1$,$0<\beta\leqslant1$的解为

$$u(t)=\int_0^t\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s-\frac{1}{2}\int_0^T\frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\cdot\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s+\frac{T^{\alpha}-2t^{\alpha}}{2\alpha T^{\alpha-1}}\int_0^T\frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s.$$

证 证明过程与文献[4]的(1.5)相类似,此处略.

2 主要结果

本节主要讨论问题(2)解的存在性.设$J=[0,T]$和$C(J)$是所有定义在J上连续实函数构成的空间.定义空间$X=\{x(t)\in C(J)\}$且赋予范数$\|x\|=\max_{t\in J}|x(t)|+\max_{t\in J}|{}^CD^qx(t)|$(其中$0<q<1$).显然,$(X,\|\cdot\|)$是 Banach 空间.

定理 1 设$f:J\times\mathbf{R}\times\mathbf{R}\to\mathbf{R}$是连续函数.假定存在 1 个常数$l\in(0,\beta]$和 1 个实值函数$m(t)\in L^{1/l}([0,T],(0,\infty))$$(l>1)$使得$|f(t,x,y)|\leqslant m(t)+d_1|x|^{\rho_1}+d_2|y|^{\rho_2}$,其中$d_1,d_2\geqslant0$,$0\leqslant\rho_1,\rho_2<1$,则问题(2)在$[0,T]$至少有 1 个解.

证 根据引理 3,问题(2)等价于如下积分方程:

$$u(t)=\int_0^t\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s-\frac{1}{2}\int_0^T\frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\cdot\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s+\frac{T^{\alpha}-2t^{\alpha}}{2\alpha T^{\alpha-1}}\int_0^T\frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,\mathrm{d}\tau-\lambda u(s)\right)\mathrm{d}s\doteq(Fu)(t).$$

定义 $B_r=\{u(t)\in X,\|u\|\leqslant r,t\in J\}$,则

$$\|{}^CD^qu(t)\|\leqslant r,$$

其中

$$r\geqslant\max\{(4Ad_1)^{1/(1-\rho_1)},(4Ad_2)^{1/(1-\rho_2)},4K,4Lr,6Br,6C,(6Dd_1)^{1/(1-\rho_1)},(6Dd_2)^{1/(1-\rho_2)}\},$$

$$A=\left(\frac{3}{2\Gamma(\alpha+\beta+1)}+\frac{1}{2\alpha\Gamma(\alpha+\beta)}\right)T^{\alpha+\beta},$$

$$B=\left(\frac{1-l}{\beta-l}\right)^{1-l}\frac{MT^{\alpha+\beta-l-q}\Gamma(\beta-l+1)}{\Gamma(\alpha+\beta+1-l-q)\Gamma(\beta)},$$

$$C=\frac{|\lambda|rT^{\alpha-q}}{\Gamma(\alpha+1-q)},$$

$$D=\left(\frac{1-l}{\beta-l}\right)^{1-l}\frac{MT^{\alpha+\beta-q}}{\Gamma(\alpha+\beta+1-q)\Gamma(\beta)},$$

$$L=|\lambda|T^{\alpha}\left(\frac{1}{\Gamma(\alpha)}+\frac{1}{\Gamma(\alpha+1)}\right),$$

$$K=\frac{\Gamma(\beta-l+1)}{\Gamma(\beta)}MT^{\alpha+\beta-l}\left(\frac{1-l}{\beta-l}\right)^{1-l}\cdot\left(\frac{3}{2\Gamma(\alpha+\beta-l+1)}+\frac{1}{2\alpha\Gamma(\alpha+\beta-l)}\right)+|\lambda|rT^{\alpha}\left(\frac{1}{\Gamma(\alpha)}+\frac{1}{\Gamma(\alpha+1)}\right),$$

$$M=\left(\int_0^T(m(s))^{1/l}\mathrm{d}s\right)^l.\qquad(3)$$

注意到B_r是 Banach 空间X的 1 个有界闭凸子集.$\forall x\in B_r$,由引理 4(Hölder 不等式)得

$$|(Fu)(t)|\leqslant\int_0^t\frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\left(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot|f(\tau,u(\tau),{}^CD^qu(\tau))|\mathrm{d}\tau\right)\mathrm{d}s+|\lambda|\|u\|\cdot$$

$$\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\mathrm{d}s + \frac{1}{2}\int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot |f(\tau,u(\tau),{}^CD^qu(\tau))|\mathrm{d}\tau\Big)\mathrm{d}s + \frac{|\lambda|\|u\|}{2}\cdot \int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\mathrm{d}s + \frac{T}{2\alpha}\Big(\int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot |f(\tau,u(\tau),{}^CD^qu(\tau))|\mathrm{d}\tau\Big)\mathrm{d}s + |\lambda|\|u\|\cdot \int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\mathrm{d}s\Big) \leqslant \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot m(\tau)\mathrm{d}\tau\mathrm{d}s + |\lambda|\|u\|\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\mathrm{d}s +$$

$$\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}(d_1r^{\rho_1}+d_2r^{\rho_2})\mathrm{d}\tau\mathrm{d}s + \frac{1}{2}\int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}m(\tau)\mathrm{d}\tau\Big)\mathrm{d}s + \frac{|\lambda|\|u\|}{2}\int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\mathrm{d}s + \frac{T}{2\alpha}\Big(\int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\cdot \Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}m(\tau)\mathrm{d}\tau\Big)\mathrm{d}s + |\lambda|\|u\|\cdot \int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\mathrm{d}s\Big) + \frac{1}{2}\int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)}\cdot$$

$$\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}(d_1r^{\rho_1}+d_2r^{\rho_2})\mathrm{d}\tau\mathrm{d}s + \frac{T}{2\alpha}\int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\cdot \Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}(d_1r^{\rho_1}+d_2r^{\rho_2})\mathrm{d}\tau\Big)\mathrm{d}s \leqslant \frac{3M}{2\Gamma(\alpha)\Gamma(\beta)}\cdot \Big(\frac{1-l}{\beta-l}\Big)^{1-l}T^{\alpha+\beta-l}\int_0^1 (1-s)^{\alpha-1}s^{\beta-l}\mathrm{d}s + \frac{(d_1r^{\rho_1}+d_2r^{\rho_2})T^{\alpha+\beta}}{\Gamma(\beta+1)}\cdot$$

$$\left(\frac{3\int_0^1 (1-s)^{\alpha-1}s^{\beta}\mathrm{d}s}{2\Gamma(\alpha)} + \frac{1}{2\alpha\Gamma(\alpha-1)}\int_0^1 (1-s)^{\alpha-2}s^{\beta}\mathrm{d}s\right) + |\lambda|rT^{\alpha}\Big(\frac{1}{\Gamma(\alpha+1)}+\frac{1}{\Gamma(\alpha)}\Big) + \frac{M}{2\alpha\Gamma(\alpha-1)\Gamma(\beta)}\cdot \Big(\frac{1-l}{\beta-l}\Big)^{1-l}T^{\alpha+\beta-l}\int_0^1 (1-s)^{\alpha-2}s^{\beta-l}\mathrm{d}s.$$

注意到 Beta 函数的性质:

$$B(\beta-l+1,\alpha) = \int_0^1 (1-s)^{\alpha-1}s^{\beta-l}\mathrm{d}s = \frac{\Gamma(\alpha)\Gamma(\beta-l+1)}{\Gamma(\alpha+\beta-l+1)},\ B(\beta+1,\alpha) = \int_0^1 (1-s)^{\alpha-1}s^{\beta}\mathrm{d}s = \frac{\Gamma(\alpha)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)},$$

$$B(\beta-l+1,\alpha-1) = \int_0^1 (1-s)^{\alpha-2}s^{\beta-l}\mathrm{d}s = \frac{\Gamma(\alpha-1)\Gamma(\beta-l+1)}{\Gamma(\alpha+\beta-l)},\ B(\beta+1,\alpha-1) = \int_0^1 (1-s)^{\alpha-2}s^{\beta}\mathrm{d}s = \frac{\Gamma(\alpha-1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta)}.$$

从而

$$|(Fu)(t)| \leqslant \frac{M\Gamma(\beta-l+1)T^{\alpha+\beta-l}}{\Gamma(\beta)}\Big(\frac{1-l}{\beta-l}\Big)^{1-l}\cdot \Big(\frac{3}{2\Gamma(\alpha+\beta-l+1)}+\frac{1}{2\alpha\Gamma(\alpha+\beta-l)}\Big) + (d_1r^{\rho_1}+d_2r^{\rho_2})T^{\alpha+\beta}\Big(\frac{3}{2\Gamma(\alpha+\beta+1)}+\frac{1}{2\alpha\Gamma(\alpha+\beta)}\Big) + \frac{|\lambda|rT^{\alpha}}{\Gamma(\alpha)}+\frac{|\lambda|rT^{\alpha}}{\Gamma(\alpha+1)},$$

$$|{}^CD^q(Fu)(t)| = \Big|\int_0^t \frac{(t-w)^{-q}}{\Gamma(1-q)}(Fu)'(w)\mathrm{d}w\Big| = \Big|\int_0^t \frac{(t-w)^{-q}}{\Gamma(1-q)}\Big(\int_0^w \frac{(w-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot f(\tau,u(\tau),{}^CD^qu(\tau))\mathrm{d}\tau - \lambda u(s)\Big)\mathrm{d}s\Big)\mathrm{d}w - \frac{1}{T^{\alpha-1}}\cdot \int_0^t \frac{(t-w)^{-q}w^{\alpha-1}}{\Gamma(1-q)}\Big(\int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot f(\tau,u(\tau),{}^CD^qu(\tau))\mathrm{d}\tau - \lambda u(s)\Big)\mathrm{d}s\Big)\mathrm{d}w\Big| \leqslant$$

$$\Big|\int_0^t \frac{(t-w)^{-q}}{\Gamma(1-q)}\Big(\int_0^w \frac{(w-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\cdot |f(\tau,u(\tau),{}^CD^qu(\tau))|\mathrm{d}\tau + |\lambda u(s)|\Big)\mathrm{d}s\Big)\mathrm{d}w - \frac{1}{T^{\alpha-1}}\int_0^t \frac{(t-w)^{-q}w^{\alpha-1}}{\Gamma(1-q)}\Big(\int_0^T \frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\cdot \Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}|f(\tau,u(\tau),{}^CD^qu(\tau))|\mathrm{d}\tau + |\lambda u(s)|\Big)\mathrm{d}s\Big)\mathrm{d}w\Big| \leqslant \int_0^t \frac{(t-w)^{-q}}{\Gamma(1-q)}\cdot$$

$$\Big(\int_0^w \frac{(w-s)^{\alpha-2}}{\Gamma(\alpha-1)\Gamma(\beta)}\Big(\int_0^s ((s-\tau)^{\beta-1})^{1/(1-l)}\mathrm{d}\tau\Big)^{1-l}\cdot \Big(\int_0^s (m(\tau))^{1/l}\mathrm{d}\tau\Big)^{l}\mathrm{d}s\Big)\mathrm{d}w + (d_1r^{\rho_1}+d_2r^{\rho_2})\cdot \int_0^t \frac{(t-w)^{-q}}{\Gamma(1-q)}\Big(\int_0^w \frac{(w-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s \frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}\mathrm{d}\tau\Big)\mathrm{d}s\Big)\mathrm{d}w +$$

$$\frac{2|\lambda|rT^{\alpha-q}}{\Gamma(\alpha+1-q)} + \frac{1}{T^{\alpha-1}}\int_0^t \frac{(t-w)^{-q}w^{\alpha-1}}{\Gamma(1-q)}\cdot \Big(\int_0^w \frac{(w-s)^{\alpha-2}}{\Gamma(\alpha-1)\Gamma(\beta)}\Big(\int_0^s ((s-\tau)^{\beta-1})^{1/(1-l)}\mathrm{d}\tau\Big)^{1-l}\cdot \Big(\int_0^s (m(\tau))^{1/l}\mathrm{d}\tau\Big)^{l}\mathrm{d}s\Big)\mathrm{d}w \leqslant 2\Big(\frac{1-l}{\beta-l}\Big)^{1-l}\cdot$$

$$\frac{MT^{\alpha+\beta-l-q}\Gamma(\beta-l+1)}{\Gamma(\alpha+\beta+1-l-q)\Gamma(\beta)} + \frac{2|\lambda|rT^{\alpha-q}}{\Gamma(\alpha+1-q)} + \Big(\frac{1-l}{\beta-l}\Big)^{1-l}\frac{M(d_1r^{\rho_1}+d_2r^{\rho_2})T^{\alpha+\beta-q}}{\Gamma(\alpha+\beta+1-q)}.$$

由(3)式得 $\|(Fu)(t)\| = \max_{t\in J}|(Fu)(t)| \leqslant r$. 注意到$(Fu)(t)$在 J 上连续. 因此,对于 $F: B_r \to B_r$,由 f 的连续性知算子 F 是 1 个全连续算子. 对于每个 $x \in B_r$,固定 $N = \max_{t\in J}|f(t,u(t),{}^CD^qu(t))|$,

$\forall \varepsilon > 0$,取

$$\delta = \frac{\varepsilon}{T^{\alpha-1}(N + 2T^{\beta}N + 3|\lambda|r)}. \tag{4}$$

对于每个 $x \in B_r$,将证明如果 $t_1, t_2 \in J$,且 $0 < t_2 - t_1 < \delta$,那么 $\|(Fu)(t_2) - (Fu)(t_1)\| < \varepsilon$.

事实上,

$$\begin{aligned}&|(Fu)(t_2) - (Fu)(t_1)| = \\ &\Big|\int_0^{t_2}\frac{(t_2-s)^{\alpha-1}}{\Gamma(\alpha)}\Big(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,d\tau - \lambda u(s)\Big)ds - \int_0^{t_1}\frac{(t_1-s)^{\alpha-1}}{\Gamma(\alpha)}\cdot \\ &\Big(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,d\tau - \lambda u(s)\Big)ds + \\ &\frac{t_1^{\alpha}-t_2^{\alpha}}{\alpha T^{\alpha-1}}\int_0^T\frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\Big(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}f(\tau,u(\tau),{}^CD^qu(\tau))\,d\tau - \lambda u(s)\Big)ds\Big| \leqslant \\ &\int_0^{t_1}\frac{(t_2-s)^{\alpha-1}-(t_1-s)^{\alpha-1}}{\Gamma(\alpha)}\Big(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}|f(\tau,u(\tau),{}^CD^qu(\tau))|\,d\tau + |\lambda u(s)|\Big)ds + \\ &\int_{t_1}^{t_2}\frac{(t_2-s)^{\alpha-1}}{\Gamma(\alpha)}\Big(\int_0^s\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}|f(\tau,u(\tau),{}^CD^qu(\tau))|\,d\tau + |\lambda u(s)|\Big)ds + \frac{t_2^{\alpha}-t_1^{\alpha}}{\alpha T^{\alpha-1}}\int_0^T\frac{(T-s)^{\alpha-2}}{\Gamma(\alpha-1)}\cdot \\ &\Big(\int_0^{t_1}\frac{(s-\tau)^{\beta-1}}{\Gamma(\beta)}|f(\tau,u(\tau),{}^CD^qu(\tau))|\,d\tau + |\lambda u(s)|\Big)ds \leqslant \frac{N(t_2^{\alpha}-t_1^{\alpha})}{\Gamma(\alpha+\beta+1)} + |\lambda|\frac{r(t_2^{\alpha}-t_1^{\alpha})}{\Gamma(\alpha+1)} + \\ &\frac{T^{\alpha+\beta-1}N}{\Gamma(\alpha)\Gamma(\beta+1)}(t_2-t_1) + \frac{T^{\alpha+\beta-1}N}{\Gamma(\alpha)}(t_2-t_1) + \frac{t_2^{\alpha}-t_1^{\alpha}}{\alpha T^{\alpha-1}}\cdot \\ &\Big(\frac{NT^{\alpha+\beta-1}}{\Gamma(\alpha+\beta)} + \frac{|\lambda|}{\Gamma(\alpha)}rT^{\alpha-1}\Big) = (t_2^{\alpha}-t_1^{\alpha})\cdot \\ &\Big[\frac{N}{\Gamma(\alpha+\beta+1)} + \frac{2|\lambda|r}{\Gamma(\alpha+1)} + \frac{NT^{\beta}}{\Gamma(\alpha+\beta)}\Big] + \\ &\frac{(t_2-t_1)}{\Gamma(\alpha)}\Big(\frac{T^{\alpha+\beta-1}N}{\Gamma(\alpha)\Gamma(\beta+1)} + |\lambda|rT^{\alpha-1}\Big).\end{aligned}$$

由平均值定理和(4)式得

$$\begin{aligned}&|(Fu)(t_2) - (Fu)(t_1)| \leqslant (t_2^{\alpha}-t_1^{\alpha})\cdot \\ &\Big[\frac{N}{\Gamma(\alpha+\beta+1)} + \frac{T^{\beta}N}{\Gamma(\alpha+\beta)} + \frac{2|\lambda|r}{\Gamma(\alpha+1)}\Big] + \\ &\frac{(t_2-t_1)}{\Gamma(\alpha)}T^{\alpha-1}\Big(\frac{T^{\beta}N}{\Gamma(\beta+1)} + |\lambda|r\Big) \leqslant (t_2-t_1)T^{\alpha-1}\cdot \\ &\Big[\frac{N}{\Gamma(\alpha+\beta+1)} + \frac{T^{\beta}N}{\Gamma(\alpha+\beta)} + \frac{2|\lambda|r}{\Gamma(\alpha+1)} + \\ &\frac{T^{\beta}N}{\Gamma(\alpha)\Gamma(\beta+1)} + \frac{|\lambda|r}{\Gamma(\alpha)}\Big] \leqslant T^{\alpha-1}(t_2-t_1)\cdot \\ &(N + T^{\beta}N + 2|\lambda|r + T^{\beta}N + |\lambda|r) < T^{\alpha-1}\delta(N + \\ &2T^{\beta}N + 3|\lambda|r) \leqslant \varepsilon,\end{aligned}$$

故有 $\|(Fu)(t_2) - (Fu)(t_1)\| < \varepsilon$.

因此,F 是同等连续和一致有界的. 由 Arzela-Ascoli 定理可知 F 在 B_r 上是紧致的,故算子 F 是全连续的. 于是算子 F 满足引理 3 的条件,由引理 3 知反周期边界值问题(2) 在 $[0,T]$ 上至少有 1 个解.

3 结论与展望

本文主要利用 Schauder 不动点定理、微积分中值定理和 Hölder 不等式等方法研究了一类非线性分数阶微分方程(2) 有解的 1 个充分条件. 在今后的研究中将讨论解的唯一性,从而为这类微分方程的数值解的计算奠定理论基础.

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The Kernel Ideal and Congruence of Pseudo Complemented MS-Algebras

ZHAO Xiu-lan,LIU Jie

(Department of Mathematics and Physics,Huang He Science and Technology College,Zhengzhou Henan 450063,China)

Abstract: The concept of kernel ideals on a pseudo complemented MS-algebras is introduced. The properties of the kernel ideals is discussed by the principal congruence of the pseudo complemented MS-algebras. The expressions of the kernel ideal congruences on the pseudo complemented MS-algebras are got.

Key words: pseudo complemented Ockham algebras; MS-algebras; kernel ideal; congruence

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The Existence of Solution to a Class of Nonlinear Fractional Differential Equation with Anti-Periodic Boundary Value Conditions

XIE Zhi-yong,XIE Xian-hua,MA Li

(School of Mathematics and Computer,Gannan Normal University,Ganzhou Jiangxi 341000,China)

Abstract: The existence of solutions for nonlinear fractional differential equation with fractional anti-periodic boundary conditions are studied. The Schauder fixed point theorem,the contraction mapping principle and Hölder inequality are applied to establish the existence.

Key words: nonlinear fractional differential equation; Schauder fixed point theorem; Hölder inequality; contraction mapping principle

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