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1 维 6 方压电准晶带 4 条裂纹的圆形孔口问题的解析解

张　峰[1],李　星[1,2*]

(1. 宁夏大学数学与计算机学院,宁夏 银川　750021;2. 宁夏师范学院数学计算机系,宁夏 固原　756000)

摘要: 利用复变函数方法与保角变换技巧,探讨了 1 维 6 方压电准晶中带有 4 条裂纹的圆形孔口的反平面 III 型裂纹问题,得出了应力强度因子和电位移强度因子的解析解. 由该解析解得出极限情形下的对称 4 裂纹圆形孔口、3 条裂纹的圆形孔口、共线双裂纹圆形孔口、单裂纹圆形孔口、十字裂纹、T 形裂纹对应的 III 型裂纹应力强度因子和电位移强度因子的解析表达式.

关键词: 1 维 6 方压电准晶; 具 4 条裂纹的圆形孔口; 复变函数方法; 应力强度因子

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0　引言与基本理论

D. Shechtman 等[1] 于 1984 年在快速冷却的铝锰合金中发现了 1 种电子衍射斑具有明显的 5 次对称性的相,并推断出该结构具有 3 维空间的彭罗斯拼图结构. D. Shechtman 等揭示出原子在晶体内的堆积形态可以不重复、独特的原子排列形式,使得固体准晶具有高硬度、低密度、耐磨等独特的性能. 因此,它成为了 1 种新型的材料,并将其广泛应用于航空航天和金属成型等工程领域[2]. 相对于经典弹性理论而言,准晶的弹性理论更复杂,应用数学与力学学者就其弹性和缺陷问题做了一系列研究. 文献[3-4] 利用对称群对于准晶的弹性基本理论开展了相关研究. 范天佑[5] 基于经典弹性理论中的复变函数法,系统地总结了若干准晶平面弹性与断裂力学问题的复变方法. 在此基础上,众多学者对简单的 1 维 6 方立方等准晶的弹性、位错、孔口和裂纹问题进行了研究[6-11]. 文献[12] 指出准晶具有压电效应,并建立了准晶的电弹性基本理论. 而考虑准晶压电效应的研究相对较少,王旭等[11] 研究了 1 维 6 方压电准晶中的动态螺旋位错,求出位错引起的应力位移场的分量的解析解. 李星等[13-14] 研究了 1 维 6 方压电准晶对称条形体中共线双半无限快速传播裂纹问题与一些简单裂纹的静力学与动力学问题的解析解.

本文运用 1 维 6 方准晶的电弹性基本理论,将文献[15] 中的具有 4 条裂纹的圆形孔口反平面问题推广到了 1 维 6 方压电准晶中,并给出了应力强度因子的解析解.

取点群 6 mm 1 维 6 方压电准晶的准周期方向为坐标轴 x_3,坐标平面为 $x_1 - x_2$ 垂直于准周期方向的平面,建立直角坐标系. 根据文献[5,11] 的广义胡克定律、几何方程、不计体力的运动平衡方程,当缺陷沿准晶的准周期方向穿透时 1 维 6 方压电准晶的反平面问题为

$$\begin{cases} C_{44}\nabla^2 u_3 + R_3\nabla^2\omega + e_{15}^1\nabla^2\psi = 0, \\ R_3\nabla^2 u_3 + K_2\nabla^2\omega + e_{15}^2\nabla^2\psi = 0, \\ e_{15}^1\nabla^2 u_3 + e_{15}^2\nabla^2\omega - \varepsilon_{44}\nabla^2\psi = 0. \end{cases} \tag{1}$$

(1) 式最终可以归结为 3 个调和函数求解的问题,由复变函数理论[5,13] 知,u_3,ω,ψ 可表示为 3 个解析函数的实部,即

$$u_3 = \mathrm{Re}\varphi_1(z),\omega = \mathrm{Re}\varphi_2(z),\psi = \mathrm{Re}\varphi_3(z), \tag{2}$$

其中 Re 表示实部,$\varphi_j(z)$ $(j = 1,2,3)$ 为 3 个解析函数,$z = x_1 + \mathrm{i}x_2$.

1　问题研究

设 1 个无限大 1 维 6 方压电准晶中有 4 条圆形孔口裂纹,裂纹长分别为 $a - R$,$b - R$,$c - R$ 和 $c - R$,裂纹沿准周期方向穿透,记边界为 L,1 个电场分布

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通信作者:李　星(1963-),男,宁夏海原人,教授,博士生导师,主要从事复分析在力学中的应用研究.

在无穷远处，电极化的方向作为准周期的方向，T 是受到的电载荷，τ 为无穷远处受到沿准周期方向的剪应力，如图 1 所示，则该问题转换为准晶在裂纹面上受到电位移和剪应力的作用，$Z = -p, Z' = -q, Z'' = -T$，电非渗透型边界条件为

$$\begin{cases} \sqrt{x_1^2 + x_2^2} \to \infty, \sigma_{31} = \sigma_{32} = H_1 = \\ H_2 = D_1 = D_2 = 0, (x_1, x_2) \in L, \\ l(\sigma_{31})_s + m(\sigma_{32})_s = -p, \\ l(H_1)_s + m(H_2)_s = -q, \\ l(D_1)_s + m(D_2)_s = -T, \end{cases} \tag{3}$$

其中 l, m 表示带 4 条裂纹的圆形孔口的外法线方向与 x_1, x_2 轴夹角的余弦值.

若 $f(z)$ 为解析函数，则 $\partial f/\partial x_1 = \mathrm{d}f/\mathrm{d}z, \partial f/\partial x_2 = \mathrm{i}\mathrm{d}f/\mathrm{d}z$.

若 $f(z) = P(x_1, x_2) + \mathrm{i}Q(x_1, x_2) = \mathrm{Re}f(z) + \mathrm{i}\mathrm{Im}f(z)$，由 Cauchy-Riemann 关系知，

$$\partial P/\partial x_1 = \partial Q/\partial x_2, \partial P/\partial x_2 = -\partial Q/\partial x_1. \tag{4}$$

由广义胡克定律和(2) 式得

$$\begin{cases} \sigma_{13} = \sigma_{31} = \\ C_{44}\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_1 + R_3\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_2 + e_{15}^1\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_3, \\ \sigma_{23} = \sigma_{32} = \\ C_{44}\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_1 + R_3\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_2 + e_{15}^1\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_3, \\ H_1 = R_3\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_1 + \kappa_2\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_2 + e_{15}^2\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_3, \\ H_2 = R_3\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_1 + \kappa_2\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_2 + e_{15}^2\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_3, \\ D_1 = e_{15}^1\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_1 + e_{15}^2\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_2 - \varepsilon_{11}\dfrac{\partial}{\partial x_1}\mathrm{Re}\varphi_3, \\ D_2 = e_{15}^1\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_1 + e_{15}^2\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_2 - \varepsilon_{11}\dfrac{\partial}{\partial x_2}\mathrm{Re}\varphi_3, \end{cases} \tag{5}$$

$$l = -\mathrm{d}x_2/\mathrm{d}s, m = \mathrm{d}x_1/\mathrm{d}s. \tag{6}$$

将(5)、(6) 式代入(3) 式，再由(4) 式知，(5) 式可化为

$$\begin{cases} \mathrm{d}(C_{44}\mathrm{Im}\varphi_1)/\mathrm{d}s + \mathrm{d}(R_3\mathrm{Im}\varphi_2)/\mathrm{d}s + \\ \mathrm{d}(e_{15}^1\mathrm{Im}\varphi_3)/\mathrm{d}s = p, \\ \mathrm{d}(R_3\mathrm{Im}\varphi_1)/\mathrm{d}s + \mathrm{d}(\kappa_2\mathrm{Im}\varphi_2)/\mathrm{d}s + \\ \mathrm{d}(e_{15}^2\mathrm{Im}\varphi_3)/\mathrm{d}s = q, \\ \mathrm{d}(e_{15}^1\mathrm{Im}\varphi_1)/\mathrm{d}s + \mathrm{d}(e_{15}^2\mathrm{Im}\varphi_2)/\mathrm{d}s - \\ \mathrm{d}(\varepsilon_{11}\mathrm{Im}\varphi_3)/\mathrm{d}s = T, \end{cases}$$

两边分别对 s 积分，并由 $\mathrm{Im}f = (f - \bar{f})/(2\mathrm{i})$ 得

$$\begin{cases} C_{44}(\varphi_1 - \bar{\varphi}_1) + R_3(\varphi_2 - \bar{\varphi}_2) + \\ e_{15}^1(\varphi_3 - \bar{\varphi}_3) = 2\mathrm{i}p\int_a^z \mathrm{d}s, \\ R_3(\varphi_1 - \bar{\varphi}_1) + \kappa_2(\varphi_2 - \bar{\varphi}_2) + \\ e_{15}^2(\varphi_3 - \bar{\varphi}_3) = 2\mathrm{i}q\int_a^z \mathrm{d}s, \\ e_{15}^1(\varphi_1 - \bar{\varphi}_1) + e_{15}^2(\varphi_2 - \bar{\varphi}_2) - \\ \varepsilon_{11}(\varphi_3 - \bar{\varphi}_3) = 2\mathrm{i}T\int_a^z \mathrm{d}s. \end{cases} \tag{7}$$

对 z 的不同位置进行讨论(见图2)，得到 $\int_a^z \mathrm{d}s$ 的结果[15] 为

$$\begin{cases} a - z, z \in \overline{AB}, \\ a - R + \int_z^R \mathrm{d}x/\sqrt{1 - (x/R)^2}, z \in \overline{BC}, \\ a - R + \pi R/2 + (z - R\mathrm{i})/\mathrm{i}, z \in \overline{CD}, \\ a - R + c - R + \pi R/2 + (c\mathrm{i} - z)/\mathrm{i}, z \in \overline{DE}, \\ a - R + 2(c - R) + \pi R/2 + \\ \int_z^{R\mathrm{i}} \mathrm{d}x/\sqrt{1 - (x/R)^2}, z \in \overline{EF}, \\ a - R + 2(c - R) + \pi R - R - z, z \in \overline{FG}, \\ a - R + 2(c - R) + (b - R) + \pi R - R - z, \\ z \in \overline{GH}, \\ a - R + 2(c - R) + 2(b - R) + \pi R + \\ \int_{-R}^z \mathrm{d}x/\sqrt{1 - (x/R)^2}, z \in \overline{HI}, \\ a - R + 2(c - R) + 2(b - R) + 3\pi R/2 + \\ (-R\mathrm{i} - z)/\mathrm{i}, z \in \overline{IJ}, \\ a - R + 3(c - R) + 2(b - R) + 3\pi R/2 + \\ (z + c\mathrm{i})/\mathrm{i}, z \in \overline{JK}, \\ a - R + 4(c - R) + 2(b - R) + 3\pi R/2 + \\ \int_{-R\mathrm{i}}^z \mathrm{d}x/\sqrt{1 - (x/R)^2}, z \in \overline{KL}, \\ a - R + 4(c - R) + 2(b - R) + 2\pi R + z - R, \\ z \in \overline{LA}. \end{cases} \tag{8}$$

(7) 式两端关于 z 求导，并将(8) 式代入得

$$\begin{cases} C_{44}(\varphi'_1 - \bar{\varphi}'_1) + R_3(\varphi'_2 - \bar{\varphi}'_2) + \\ e_{15}^1(\varphi'_3 - \bar{\varphi}'_3) = 2\mathrm{i}pu(z), \\ R_3(\varphi'_1 - \bar{\varphi}'_1) + \kappa_2(\varphi'_2 - \bar{\varphi}'_2) + \\ e_{15}^2(\varphi'_3 - \bar{\varphi}'_3) = 2\mathrm{i}qu(z), \\ e_{15}^1(\varphi'_1 - \bar{\varphi}'_1) + e_{15}^2(\varphi'_2 - \bar{\varphi}'_2) - \\ \varepsilon_{11}(\varphi'_3 - \bar{\varphi}'_3) = 2\mathrm{i}Tu(z), \end{cases} \tag{9}$$

其中

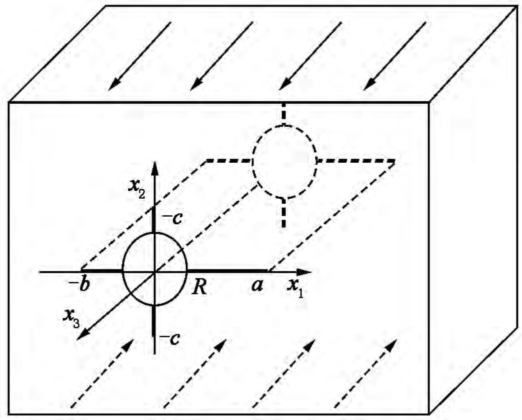

图 1 具有 4 条裂纹的圆形孔口模型

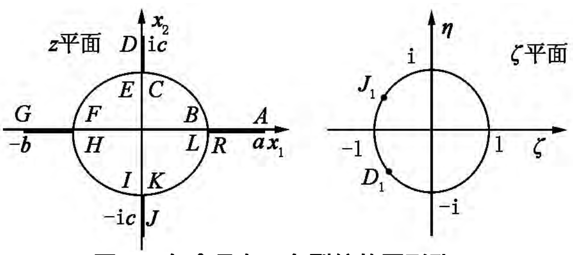

图 2 包含具有 4 条裂纹的圆形孔口的无限大平面到单位圆的映射

$$u(z)=\begin{cases}1, & z\in\overline{GH}\cup\overline{LA},\\ -1, & z\in\overline{AB}\cup\overline{FG},\\ \mathrm{i}, & z\in\overline{DE}\cup\overline{IJ},\\ -\mathrm{i}, & z\in\overline{CD}\cup\overline{JK},\\ 1/\sqrt{1-(z/a)^2}, & z\in\overline{HI}\cup\overline{KL},\\ -1/\sqrt{1-(z/a)^2}, & z\in\overline{BC}\cup\overline{EF}.\end{cases}$$

引入保角变换[15]

$$z=\omega(\zeta)=R\{\sqrt{-16c_1^2\zeta^2+X}+\sqrt{-16c_1^2\zeta^2-16\zeta^2+X}\}/(4\xi),$$

其中 $X=\left[\sqrt{b_1^2+c_1^2}(\zeta-1)^2+\sqrt{a_1^2+c_1^2}(\zeta+1)^2\right]^2$，$a_1=(a/R+R/a)/2$，$b_1=(b/R+R/b)/2$，$c_1=(c/R+R/c)/2$．此映射将 z 平面的裂纹映射到 ζ 平面上的单位圆内部，从而 $\omega^{-1}(a)\to 1$，$\omega^{-1}(-b)\to -1$，$\omega^{-1}(\mathrm{i}c)\to D_1$，$\omega^{-1}(-\mathrm{i}c)\to J_1$，如图 2 所示.

令 $\psi_i(\zeta)=\varphi_i(z)$，则

$$\varphi'_i(z)=\psi'_i(\zeta)/\omega'(\zeta)\ (i=1,2,3).\tag{10}$$

将(10)式代入(9)式整理，然后将单位圆上的点 $\zeta=\sigma=\mathrm{e}^{\mathrm{i}\theta}$ 代入，得

$$\begin{cases}\psi'_1(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_1(\sigma)}+\dfrac{R_3}{C_{44}}\left[\psi'_2(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_2(\sigma)}\right]+\dfrac{e_{15}^1}{C_{44}}\left[\psi'_3(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_3(\sigma)}\right]=\dfrac{2p\mathrm{i}}{C_{44}}\omega'(\sigma)u(\sigma),\\ \dfrac{R_3}{k_2}\left[\psi'_1(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_1(\sigma)}\right]+\left[\psi'_2(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_2(\sigma)}\right]+\dfrac{e_{15}^2}{k_2}\left[\psi'_3(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_3(\sigma)}\right]=\dfrac{2q\mathrm{i}}{k_2}\omega'(\sigma)u(\sigma),\\ \dfrac{e_{15}^1}{\varepsilon_{11}}\left[\psi'_1(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_1(\sigma)}\right]+\dfrac{e_{15}^2}{\varepsilon_{11}}\left[\psi'_2(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_2(\sigma)}\right]-\left[\psi'_3(\sigma)-\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'_3(\sigma)}\right]=\dfrac{2T\mathrm{i}}{\varepsilon_{11}}\omega'(\sigma)u(\sigma),\end{cases}\tag{11}$$

(11)式两边分别同时乘以 $1/[2\pi\mathrm{i}(\sigma-\xi)]$，并沿边界 l 积分得

$$\begin{cases}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_1(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_1(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]+\dfrac{R_3}{C_{44}}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_2(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_2(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]+\dfrac{e_{15}^1}{C_{44}}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_3(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_3(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]=\dfrac{2p\mathrm{i}}{C_{44}}\dfrac{1}{2\pi\mathrm{i}}\int_l\dfrac{\omega'(\sigma)u(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma,\\ \dfrac{R_3}{k_2}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_1(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_1(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]+\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_2(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_2(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]+\dfrac{e_{15}^2}{k_2}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_3(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_3(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]=\dfrac{2q\mathrm{i}}{k_2}\dfrac{1}{2\pi\mathrm{i}}\int_l\dfrac{\omega'(\sigma)u(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma,\\ \dfrac{e_{15}^1}{\varepsilon_{11}}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_1(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_1(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]+\dfrac{e_{15}^2}{\varepsilon_{11}}\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_2(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_2(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]-\dfrac{1}{2\pi\mathrm{i}}\left[\oint_l\dfrac{\psi'_3(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma-\int_l\dfrac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\dfrac{\overline{\psi'_3(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\right]=\dfrac{2T\mathrm{i}}{\varepsilon_{11}}\int_l\dfrac{\omega'(\sigma)u(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma.\end{cases}\tag{12}$$

在单位圆内 $\psi'_i(\zeta)$ 解析，由 Cauchy 积分公式得

$$\frac{1}{2\pi i}\int_l \frac{\psi'_i(\sigma)}{\sigma-\zeta}d\sigma = \psi'_i(\zeta).$$

由于 $\omega'(\sigma)/\overline{\omega'(\sigma)} = -1/\sigma^2$，则 $\omega'(\sigma)\cdot\overline{\psi'_i(\sigma)}/\overline{\omega'(\sigma)}$ 为单位圆外解析函数 $-\overline{\psi'_i}(1/\zeta)/\zeta^2$ 的边值. 根据无穷远处的 Cauchy 积分公式，对于 $|\zeta|<1$，有

$$\frac{1}{2\pi i}\int_l \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'_i(\sigma)}}{\sigma-\zeta}d\sigma = 0.$$

令 $F(\zeta) = \dfrac{1}{2\pi i}\displaystyle\int_l \frac{\omega'(\sigma)u(\sigma)}{\sigma-\zeta}d\sigma$，其中

$$F(\zeta) = \begin{cases} \pm RY/2, & z\in \overline{AB}\cup\overline{FG}\cup\overline{GH}\cup\overline{LA}, \\ \pm iRY/2, & z\in \overline{CD}\cup\overline{DE}\cup\overline{IJ}\cup\overline{JK}, \\ 0, & z\in \overline{BC}\cup\overline{EF}\cup\overline{HI}\cup\overline{KL}, \end{cases}$$

其中 $Y = \sqrt{a_1^2 + b_1^2 + 2(c_1^2 + \sqrt{a_1^2+c_1^2}\sqrt{b_1^2+c_1^2})}$.

由(12)式可解得

$$\begin{cases} \psi'_1(\zeta) = 2i\,[e_{15}^1k_2T + (e_{15}^2)^2p + \varepsilon_{11}k_2p - R_3\varepsilon_{11}q - R_3e_{15}^2T - e_{15}^1e_{15}^2q]F(\zeta)\,/\,[C_{44}(e_{15}^2)^2 - 2e_{15}^1e_{15}^2R_3 + k_2(e_{15}^1)^2 - R_3^2\varepsilon_{11} + C_{44}\varepsilon_{11}k_2], \\ \psi'_2(\zeta) = 2i\,[(e_{15}^1)^2q + C_{44}\varepsilon_{11}q - R_3\varepsilon_{11}p + C_{44}e_{15}^2T - e_{15}^1e_{15}^2p - e_{15}^1R_3T]F(\zeta)\,/\,[C_{44}(e_{15}^2)^2 - 2e_{15}^1e_{15}^2R_3 + k_2(e_{15}^1)^2 - R_3^2\varepsilon_{11} + C_{44}\varepsilon_{11}k_2], \\ \psi'_3(\zeta) = 2i\,[R_3^2T + e_{15}^1k_2p - R_3e_{15}^1q - R_3e_{15}^2p + C_{44}e_{15}^2q - C_{44}k_2T]F(\zeta)\,/\,[C_{44}(e_{15}^2)^2 - 2e_{15}^1e_{15}^2R_3 + k_2(e_{15}^1)^2 - R_3^2\varepsilon_{11} + C_{44}\varepsilon_{11}k_2]. \end{cases} \quad (13)$$

根据(4)式和(5)式得

$$\begin{cases} \sigma_{31} - i\sigma_{32} = C_{44}\varphi'_1 + R_3\varphi'_2 + e_{15}^1\varphi'_3, \\ H_1 - iH_2 = R_3\varphi'_1 + k_2\varphi'_2 + e_{15}^2\varphi'_3, \\ \sigma_{31} - i\sigma_{32} = e_{15}^1\varphi'_1 + e_{15}^2\varphi'_2 - \varepsilon_{11}\varphi'_3. \end{cases} \quad (14)$$

由(13)式代入(10)式可得 $\varphi'_i(z)$，然后代入(14)式解方程组，再利用保角映射的反演，则可得 z 平面内的应力场和位移场的表达式，由于结果较复杂，此处不予列出.

由 $1/\sqrt{\omega''(\zeta)} \to \sqrt{2a_1\sqrt{a_1^2-1}}/\sqrt{PR}$，$\zeta\to 1(z\to a)$，$F(\zeta)\to RY/2$，$\zeta\to 1(z\to a)$ 得在裂纹尖端 $z=a$ 处，对应于 ζ 平面内 $\zeta=1$ 处的声子场应力强度因子、相位子场应力强度因子、电位移强度因子分别为

$$K_{III}^{II} = \lim_{\zeta\to 1}2\sqrt{\pi}pF(\zeta)/\sqrt{\omega''(\zeta)} = pF(R),$$
$$K_{III}^{\perp} = \lim_{\zeta\to 1}2\sqrt{\pi}qF(\zeta)/\sqrt{\omega''(\zeta)} = qF(R), \quad (15)$$
$$K_{III}^{D} = \lim_{\zeta\to 1}2\sqrt{\pi}TF(\zeta)/\sqrt{\omega''(\zeta)} = TF(R),$$

其中

$$P = (a_1 + \sqrt{a_1^2-1})(a_1^2+c_1^2) + \sqrt{a_1^2+c_1^2}\sqrt{b_1^2+c_1^2},$$
$$F(R) = \sqrt{2\pi R}\sqrt{a_1\sqrt{a_1^2-1}}Y/\sqrt{P}.$$

当 1 维 6 方压电准晶退化为 1 维 6 方准晶时，问题就退化为 1 维 6 方准晶中具有 4 条裂纹的圆形孔口，其声子场应力、相位子场应力强度因子分别为

$$K_{III}^{II} = \lim_{\zeta\to 1}2\sqrt{\pi}pF(\zeta)/\sqrt{\omega''(\zeta)} = pF(R),$$
$$K_{III}^{\perp} = \lim_{\zeta\to 1}2\sqrt{\pi}qF(\zeta)/\sqrt{\omega''(\zeta)} = qF(R),$$

与文献[15]中的结果一致.

2　几种特殊情形

1)(i) 当 $c\to R$ 时，问题为 1 维 6 方压电准晶中带非对称共线双裂纹的圆形孔口，由(15)式得

$$K_{III}^{II} = pG(R),\ K_{III}^{\perp} = qG(R),\ K_{III}^{D} = TG(R), \quad (16)$$

其中

$$G(R) = \sqrt{2\pi(a+b)(R^2+ab)(a^2-R^2)/(ab)}/(2a).$$

(ii) 当 $b\to a$ 时，问题为 1 维 6 方压电准晶中带有对称共线双裂纹的圆形孔口，由(16)式知

$$K_{III}^{II} = p\sqrt{\pi a(1-R^4/a^4)},\ K_{III}^{\perp} = q\sqrt{\pi a(1-R^4/a^4)},$$
$$K_{III}^{D} = T\sqrt{\pi a(1-R^4/a^4)}.$$

(iii) 当 $b\to R$ 时，问题为1维6方压电准晶中带有单裂纹的圆形孔口，由(16)式知

$$K_{III}^{II} = p\sqrt{2a\pi}(a+R)\sqrt{a^2-R^2}/(2a^2),$$
$$K_{III}^{\perp} = q\sqrt{2a\pi}(a+R)\sqrt{a^2-R^2}/(2a^2),$$
$$K_{III}^{D} = T\sqrt{2a\pi}(a+R)\sqrt{a^2-R^2}/(2a^2).$$

2) 当 $b\to a$ 时，问题为 1 维 6 方压电准晶中具有对称 4 裂纹的圆形孔口，由(15)式知

$$K_{III}^{II} = p\sqrt{\pi a(1-R^4/a^4)},\ K_{III}^{\perp} = q\sqrt{\pi a(1-R^4/a^4)},$$
$$K_{III}^{D} = T\sqrt{\pi a(1-R^4/a^4)}.$$

3) 当 $b\to R$ 时，问题为 1 维 6 方压电准晶中具有 3 条裂纹的圆形孔口，由(15)式知

$$K_{III}^{II} = pH(R),\ K_{III}^{\perp} = qH(R),\ K_{III}^{D} = TH(R),$$

其中

$$H(R) = \sqrt{(L_1+L_2)/(L_3+L_4)}M,$$
$$L_1 = 2a^2c^2R^2 + a^4c^2 + c^2R^4,\ L_2 = 2a(c^2+R^2)\cdot\sqrt{(a^2+c^2)(a^2c^2+R^4)} + 2a^2(a^4+R^4),$$
$$L_3 = a^4c^2 + c^2R^4,\ L_4 = a^2[c^4 + R^4 + c(c^2+R^2)\cdot\sqrt{a^2+c^2+R^4/a^2+R^4/c^2}],$$
$$M = \sqrt{\pi c^2(a^4-R^4)/(2a^3c^2)}.$$

4)(i) 当 $R\to 0$ 时，问题为 1 维 6 方压电准晶中具有非对称十字裂纹的圆形孔口，由(15)式知

$$\begin{cases} K_{III}^{II} = \frac{p\sqrt{2\pi a}}{2}\sqrt{1+\frac{\sqrt{b^2+c^2}}{\sqrt{a^2+c^2}}}, \\ K_{III}^{\perp} = \frac{q\sqrt{2\pi a}}{2}\sqrt{1+\frac{\sqrt{b^2+c^2}}{\sqrt{a^2+c^2}}}, \\ K_{III}^{D} = \frac{T\sqrt{2\pi a}}{2}\sqrt{1+\frac{\sqrt{b^2+c^2}}{\sqrt{a^2+c^2}}}. \end{cases} \tag{17}$$

(ii) 当 $b \to a$ 时,问题为 1 维 6 方压电准晶中具有对称十字裂纹的圆形孔口,由(17)式知

$$K_{III}^{II} = p\sqrt{\pi a}, K_{III}^{\perp} = q\sqrt{\pi a}, K_{III}^{D} = T\sqrt{\pi a}.$$

(iii) 当 $b \to 0$ 时,问题为 1 维 6 方压电准晶中 T 形裂纹,由(17)式知

$$K_{III}^{II} = \frac{p\sqrt{2\pi a}}{2}\sqrt{1+\frac{c}{\sqrt{a^2+c^2}}}, K_{III}^{\perp} = \frac{q\sqrt{2\pi a}}{2}\cdot\sqrt{1+\frac{c}{\sqrt{a^2+c^2}}}, K_{III}^{D} = \frac{T\sqrt{2\pi a}}{2}\sqrt{1+\frac{c}{\sqrt{a^2+c^2}}}.$$

(iv) 当 $b \to a, c \to 0$ 时,问题为 1 维 6 方压电准晶中 Griffith 裂纹,由(17)式知

$$K_{III}^{II} = p\sqrt{\pi a}, K_{III}^{\perp} = q\sqrt{\pi a}, K_{III}^{D} = T\sqrt{\pi a}.$$

当 1 维 6 方压电准晶退化为 1 维 6 方准晶时,问题为 1 维 6 方准晶中的 Griffith 裂纹,声子场应力、相位子场应力强度因子分别为 $K_{III}^{II} = p\sqrt{\pi a}$, $K_{III}^{\perp} = q\cdot\sqrt{\pi a}$. 与文献[15]中的结果一致.

3 结论

本文得出了 1 维 6 方压电准晶中具有 4 条裂纹的圆形孔口问题的应力和电位移强度因子的解析解,从得到的结果可以看出:电位移并没有和应力发生耦合,电载荷不能改变应力场,只能改变电位移应力强度因子,声子场应力、相位子场应力以及电位移都受到裂纹尺寸的影响. 当 1 维 6 方压电准晶退化为 1 维 6 方准晶时,与文献[15]的结论一致. 得到的解析解在极限情形下可以给出 1 维 6 方压电准晶中对称 4 裂纹圆形孔口、3 条裂纹的圆形孔口、共线双裂纹圆形孔口、单裂纹圆形孔口、十字裂纹、T 形裂纹问题对应的 III 型裂纹应力强度因子和电位移强度因子的解析表达式. 这在一定意义上验证理论推导的正确性. 在自然界和工程应用中经常会遇到带裂纹的材料受到电载荷的作用,材料由裂纹尖端开始破坏是常有的,本文得到的结果可以作为理论依据,从而推进压电准晶材料的实际应用.

4 参考文献

[1] Shechtman D, Blech I, Gratias D, et al. Metallic phase with long-range orientational order and no translational symmetry [J]. Physical Review Letters, 1984, 53(20): 1951-1953.

[2] Dubois J M. So useful, those quasicrystals [J]. Israel Journal of Chemistry, 2011, 51(11/12): 1168-1175.

[3] Hu Chengzheng, Wang Renhui, Ding Dihua. Symmetry groups, physical property tensors, elasticity and dislocations in quasicrystals [J]. Rep Prog Phys, 2000, 63(1): 1-39.

[4] Ding Dihua, Wang Renhui, Yang Wenge. Generalized elasticity theory of quasicrystals [J]. Physical Review B, 1993, 48(10): 7003-7010.

[5] Fan Tianyou. Mathematical theory of elasticity of quasicrystals and its applications [M]. Beijing: Science Press, 2011.

[6] Li Xianfang, Fan Tianyou. A straight dislocation in one-dimensional hexagonal quasicrystals [J]. Physica Status Solidi: B, 1999, 212(1): 19-26.

[7] Peng Yanze, Fan Tianyou. Crack and indentation problems for one-dimensional hexagonal quasicrystals [J]. The European Physical Journal B-Condensed Matter and Complex Systems, 2001, 21(1): 39-44.

[8] Chen Weiqin, Ma Yingliang, Ding Haojiang. On three-dimensional elastic problems of one-dimensional hexagonal quasicrystal bodies [J]. Mechanics Research Communications, 2004, 31(6): 633-641.

[9] Liu Guanting, Fan Tianyou, Guo Ruiping. Governing equations and general solutions of plane elasticity of one-dimensional quasicrystals [J]. International Journal of Solids and Structures, 2004, 41(14): 3949-3959.

[10] Guo Junhong, Liu Guanting. Analytic solutions to problem of elliptic hole with two straight cracks in one-dimensional hexagonal quasicrystals [J]. Applied Mathematics and Mechanics, 2008, 29(4): 485-493.

[11] Wang Xu. The general solution of one-dimensional hexagonal quasicrystal [J]. Mechanics Research Communications, 2006, 33(4): 576-580.

[12] Hu Chengzheng, Wang Renhui, Ding Dihua. Piezoelectric effects in quasicrystals [J]. Physical Review B, 1997, 56(5): 2463.

[13] 李星, 霍华颂, 时朋朋. 一维六方压电准晶对称条形体中共线双半无限快速传播裂纹的解析解[J]. 固体力学学报, 2014, 35(2): 135-141.

[14] 霍华颂. 一维六方压电准晶缺陷的静力学与动力学问题[D]. 宁夏: 宁夏大学, 2014.

[15] 陈柱. 断裂力学中具有多条裂纹的圆形孔口问题的解析研究[D]. 呼和浩特: 内蒙古师范大学, 2008.

(下转第 100 页)

The Comprehensive Evaluation about Algal Growth Adaptability in Swine Wastewater Based on Factor Analysis of SPSS Software

HUANG Xueping[1,2], LI Wei[3], WAN Jinbao[1*], KE Ying[4], HONG Tao[2], XIE Youlin[2], CHEN Xiaoting[2]

(1. Key Lab of Poyang Lake Environment and Resource Utilization, Ministry of Education, School of Environment and Chemical Engineering, Nanchang University, Nanchang Jiangxi 330047, China;
2. Deptment of Civil Engineering and Architecture, Nanchang Institute of Technology, Jiangxi Provincial Engineering Research Center of the Special Reinforcement and Safety Monitoring Technology in Hydraulic & Civil Engineering, Nanchang Jiangxi 330099, China;
3. Economic and Trade Department, Jiangxi Biotech Vocational College, Nanchang Jiangxi 330200, China;
4. Journal Editorial Department, Nanchang Institute of Technology, Nanchang Jiangxi 330099, China)

Abstract: Based on factor analysis of SPSS software, it was researched that the growth adaptability of three kinds algae such as *Chlorella*, *Scenedesmus* and *Microcystis* in eight differentstages of swine wastewater treatment. It aimed at making a certain exploration to analysis and evaluation about the adaptability of algal growth in swine wastewater and the possibility of pig wastewater as algae culture medium. The research results showed that in all the eight wastewater samples, the raw wastewater's nutrition is more suitable for algal growth, and *Chlorella* grow strongest in swine wastewater and can become dominant species.

Key words: SPSS; factor analysis; swine wastewater; algae growth adaptability; *Chlorella*

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(上接第 54 页)

The Analytic Solutions of a Circular Hole with Four Cracks of One-Dimensional Hexagonal Piezoelectric Quasicrystals

ZHANG Feng[1], LI Xing[1,2*]

(1. School of Mathematics and Computer Science, Ningxia University, Yinchuan Ningxia 750021, China;
2. School of Mathematics and Computer Science, Ningxia Teachers University, Guyuan Ningxia 756000, China)

Abstract: By using the complex variable function method and the conformal mapping technique, the antiplane problem of one-dimensional hexagonal piezoelectric quasicrystals with a circular hole with four cracks is analyzed. The solutions of the stress intensity factors and electric displacement intensity factor is given in analytical form. For some special cases, such as a circular hole with symmetrical four cracks, a circular hole with three cracks, a circular hole with two collinear cracks, a circular hole with a single crack, a cross crack and a T shape crack in one-dimensional hexagonal piezoelectric quasicrystals, the analytic expressions of the stress intensity factor and the electric displacement intensity factor can be obtained by taking a limiting process from our present solution.

Key words: one-dimensional hexagonal piezoelectric quasicrystal; an circular hole with four cracks; complex variable method; SIFs

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