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Lie Symmetry and Conserved Quantity Based on El-Nabulsi Models in Phase Space

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Abstract: In phase space, the Lie symmetry and conserved quantity for non-conservative dynamics based on El-Nabulsi models are studied. Firstly, the differential equations of motion of the systems are established. Secondly, the determining equations are established in phase space under a general infinitesimal transformation, thus the definition and the criterion of Lie symmetry based on El-Nabulsi models are obtained. At the same time, the form of generalized Hojman conserved quantity as a direct result of the Lie symmetry is given in phase space, and the Hojman conserved quantity acts as a special case of the generalized Hojman conserved quantity. Then, the Noether conserved quantity of the Lie symmetry based on El-Nabulsi models is gained. Lastly, two examples are given to illustrate the application of the results.

Key words: El-Nabulsi models; in phase space; the Lie symmetry; generalized Hojman conserved quantity; Noether conserved quantity

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0 Introdution

In 1918, Noether which put forward the famous theorem revealed the relationship between the symmetry and conserved quantity. But the Noether symmetry did not include all the symmetries. In 1979, M. Lutzky [1] applied Sophus Lie's invariance theory of differential equations into the field of dynamics, and discussed the Lie symmetry and conserved quantity of dynamic systems. The Lie symmetry showed its unique advantages in basic subjects and practical applications, and made outstanding contributions to the similar analysis of mechanical systems and the nonlinear mechanics. Later then, the Lie symmetry had been widely concerned. In 1994, Zhao Yueyu [2] extended the Lie symmetry theory into non-conservative mechanical systems, got its Noether conserved quantity. Mei Fengxiang [3] comprehensively and systematically studied the Lie symmetry and Noether conserved quantity of constrained mechanical systems. In 1992, S. A. Hojman [4] presented a direct method of finding conserved quantity by the symmetry, and the conserved quantity is called Hojman conserved quantity. And its characteristic is that it could directly get through infinitesimal transformations of group to find the conserved quantity. After this, great achievements about Hojman conserved quantity and Noether conserved quantity by the Lie symmetry have been gained [5-10].

In 1695, L'Hopital first mentioned the concept of fractional calculus in the letter to Leibnitz. In 1996, F. Riewe [11] first researched the fractional variational problems, and applied the fractional calculus to non-conservative modeling. Since then, Klimek, Agrawal, Atanackovic, Torres, El-Nabulsi and many other scholars took a series of fruitful results in this respect [12-16]. In 2005, El-Nabulsi [16] proposed a new modeling method based on non-conservative dynamical systems,

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namely El-Nabulsi′s dynamics model. In this method, the fractional integral about time only needs one real parameter, the equations established under El-Nabulsi′s dynamics model were similar to the classical equation of motion and didn′t contain the fractional derivative. Subsequently, El-Nabulsi expanded this method to the situation of Lagrangian depending on fractional derivatives, and studied the multi-dimensional function variational problems, and researched the variational problems with holonomic constrains or nonholonomic constrains or dissipative dynamic systems [17-19]. Frederico [15,20] studied the Noether′s theorem in non-conservative mechanics, and extended to the situation of Lagrangian containing higher-order derivatives. Zhang Yi, et al [21-23] carried out the study of the variational problems and symmetry with mechanical systems based on El-Nabulsi models. In this paper, we will further study the Lie symmetry based on El-Nabulsi models in phase space, and give the generalized Hojman conserved quantity and the Noether conserved quantity.

1 Differential Equations of Motion of the Systems

If the configuration of a mechanical system is determined by the generalized coordinates $q_s(s=1,2,\cdots,n)$, where $L=L(\tau,q_s,\dot{q}_s)$ is the Lagrangian of the system. At the same time, L is a function to be C^2-smooth, namely $L\in C^m(\mathbf{R}^{2n+1})$, and the system is nonsingular, that is $\det(\partial^2 L/(\partial\dot{q}_s\partial\dot{q}_k))\neq 0$. Introduce the generalized momentum and the Hamiltonian

$$p_s=\partial L/\partial\dot{q}_s, H=p_s\dot{q}_s-L=H(\tau,q_s,p_s). \quad (1)$$

Base on the definition of left Riemann-Liouville fractional derivative [16], El-Nabulsi put forward a class of fractional variational problems as follows

$$ {}_0I_t^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t f(\tau)(t-\tau)^{\alpha-1}\mathrm{d}\tau. \quad (2)$$

Find the stationary points of the integral functional

$$S(\gamma)=\frac{1}{\Gamma(\alpha)}\int_{\tau_1}^{\tau_2}[p_s\dot{q}_s-H(\tau,q_s,p_s)](t-\tau)^{\alpha-1}\mathrm{d}\tau \quad (3)$$

in the fixed boundary conditions $q_s(\tau_1)=q_{s,1}, q_s(\tau_2)=q_{s,2}(s=1,2,\cdots,n)$, where γ is one curve, $0<\alpha\leqslant 1$, $\dot{q}_s=\mathrm{d}q_s/\mathrm{d}\tau$, and Γ is the Euler-Gamma function, t is the observer time, τ is the intrinsic time, $t\neq\tau$, and the Hamiltonian H is a C^2-function.

According to the theory of the calculus of variations, the necessary condition for the functional (3) having an extreme value at $q_s=q_s(\tau)$, $p_s=p_s(\tau)$ is that its variation equals to zero, namely $\delta S=0$, so one can obtain the fractional Hamilton canonical equations based on El-Nabulsi models [21]

$$\dot{q}_s=\frac{\partial H}{\partial p_s}, \dot{p}_s=-\frac{\partial H}{\partial q_s}-\frac{1-\alpha}{t-\tau}p_s(s=1,2,\cdots,n). \quad (4)$$

When α is equal to 1, equations (4) is the classical Hamilton canonical equations.

2 The Lie Symmetry and Generalized Hojman Conserved Quantity

2.1 The Infinitesimal Transformations and Determining Equations of the Lie Symmetry

Introduce the infinitesimal group transformations with respect to time τ

$$\tau^*=\tau+\Delta\tau, q_s^*(\tau^*)=q_s(\tau)+\Delta q_s,$$
$$p_s^*(\tau^*)=p_s(\tau)+\Delta p_s, s=1,2,\cdots,n, \quad (5)$$

or their expansion formulae

$$\begin{cases}\tau^*=\tau+\varepsilon\xi_0(\tau,q_k,p_k),\\ q_s^*(\tau^*)=q_s(\tau)+\varepsilon\xi_s(\tau,q_k,p_k),\\ p_s^*(\tau^*)=p_s(\tau)+\varepsilon\eta_s(\tau,q_k,p_k),\\ s,k=1,2,\cdots,n,\end{cases} \quad (6)$$

where ε is an infinitesimal parameter, ξ_0, ξ_s and η_s are functions of the infinitesimal transformations.

Let us introduce the infinitesimal generator

$$X^{(0)}=\xi_0\frac{\partial}{\partial\tau}+\xi_s\frac{\partial}{\partial q_s}+\eta_s\frac{\partial}{\partial p_s}, \quad (7)$$

and its first extension is

$$X^{(1)}=X^{(0)}+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_s-\dot{q}_s\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0\right)\frac{\partial}{\partial\dot{q}_s}+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\eta_s-\dot{p}_s\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0\right)\frac{\partial}{\partial\dot{p}_s}(s=1,2,\cdots,n), \quad (8)$$

where $\bar{d}/\mathrm{d}\tau=\partial/\partial\tau+\dot{q}_s\partial/\partial q_s+\dot{p}_s\partial/\partial p_s=\partial/\partial\tau+(\partial H/\partial p_s\cdot\partial/\partial q_s-(\partial H/\partial q_s+(1-\alpha/(t-\tau)p_s)\partial/\partial p_s(s=1,2,\cdots,n)$.

According to the invariance theory of differential equations under infinitesimal transformations, the invariance of equations (4) comes down to

$$X^{(1)}\left[\dot{q}_s-\partial H/\partial p_s\right]=0,$$
$$X^{(1)}\left[\dot{p}_s+\partial H/\partial q_s+(1-\alpha)/(t-\tau)p_s\right]=0,\quad(9)$$

if

$$\dot{q}_s=\frac{\partial H}{\partial p_s},\dot{p}_s=-\frac{\partial H}{\partial q_s}-\frac{1-\alpha}{t-\tau}p_s(s=1,2,\cdots,n).\quad(10)$$

Substituting operator (8) into formula (9) and taking notice of the equations(10) can obtain

$$\frac{\bar{d}}{d\tau}\xi_s-\frac{\partial H}{\partial p_s}\frac{\bar{d}}{d\tau}\xi_0=X^{(0)}\left(\frac{\partial H}{\partial p_s}\right),\frac{\bar{d}}{d\tau}\eta_s+\left(\frac{\partial H}{\partial q_s}+\frac{1-\alpha}{t-\tau}p_s\right)\frac{\bar{d}}{d\tau}\xi_0=X^{(0)}\left(-\frac{\partial H}{\partial q_s}-\frac{1-\alpha}{t-\tau}p_s\right),(s=1,2,\cdots,n).\quad(11)$$

Equations (11) are called the determining equations with the Lie symmetry.

Definition 1　If the generators of infinitesimal transformations (6) satisfy the determining equations (11), the corresponding symmetry is called the Lie symmetry of non-conservative dynamical system under El-Nabulsi model in phase space.

2.2　The Generalized Hojman Theorem

The Lie symmetry does not always lead to the conserved quantity. The existence condition and the form of generalized Hojman conserved quantity of the Lie symmetry under El-Nabulsi model in phase space will be given in the following theorem.

Theorem 1 For non-conservative dynamical system (4) under El-Nabulsi model in phase space, if the infinitesimal generators ξ_0,ξ_s,η_s satisfy the determining equations (11), and there exists with a function $\lambda=\lambda(t,q_s,p_s)$ admitting

$$\frac{\partial}{\partial q_s}\frac{\partial H}{\partial p_s}-\frac{\partial}{\partial p_s}\left(\frac{\partial H}{\partial q_s}+\frac{1-\alpha}{t-\tau}p_s\right)+\frac{\bar{d}}{d\tau}\ln\lambda=0.\quad(12)$$

Then, the Lie symmetry of the system directly leads to the generalized Hojman conserved quantity, that is

$$I=\frac{1}{\lambda}\frac{\partial}{\partial\tau}(\lambda\xi_0)+\frac{1}{\lambda}\frac{\partial}{\partial q_s}(\lambda\xi_s)+\frac{1}{\lambda}\frac{\partial}{\partial p_s}(\lambda\eta_s)-\frac{\bar{d}}{d\tau}\xi_0=\text{const}.\quad(13)$$

Proof

$$\frac{\bar{d}}{d\tau}I=\frac{\bar{d}}{d\tau}\frac{\partial\xi_0}{\partial\tau}+\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\frac{\partial\lambda}{\partial\tau}\xi_0\right)+\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\frac{\partial\lambda}{\partial q_s}\xi_s\right)+\frac{\bar{d}}{d\tau}\frac{\partial\xi_s}{\partial q_s}+\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\frac{\partial\lambda}{\partial p_s}\eta_s\right)+\frac{\bar{d}}{d\tau}\frac{\partial\eta_s}{\partial p_s}-\frac{\bar{d}}{d\tau}\frac{\bar{d}}{d\tau}\xi_0.\quad(14)$$

It´s easy to obtain

$$\begin{cases}\frac{\bar{d}}{d\tau}\frac{\partial\xi_0}{\partial\tau}=\frac{\partial}{\partial\tau}\frac{\bar{d}}{d\tau}\xi_0-\frac{\partial\dot{q}_s}{\partial\tau}\frac{\partial\xi_0}{\partial q_s}-\frac{\partial\dot{p}_s}{\partial\tau}\frac{\partial\xi_0}{\partial p_s},\\ \frac{\bar{d}}{d\tau}\frac{\partial\xi_s}{\partial q_s}=\frac{\partial}{\partial q_s}\frac{\bar{d}}{d\tau}\xi_s-\frac{\partial\dot{q}_s}{\partial q_s}\frac{\partial\xi_s}{\partial q_s}-\frac{\partial\dot{p}_s}{\partial q_s}\frac{\partial\xi_s}{\partial p_s},\\ \frac{\bar{d}}{d\tau}\frac{\partial\eta_s}{\partial p_s}=\frac{\partial}{\partial p_s}\frac{\bar{d}}{d\tau}\eta_s-\frac{\partial\dot{q}_s}{\partial p_s}\frac{\partial\eta_s}{\partial q_s}-\frac{\partial\dot{p}_{s\ s}}{\partial p_s}\frac{\partial\eta_s}{\partial p_s}.\end{cases}\quad(15)$$

Substituting formulae (15) into formula (14) and using the determining equations (11), then have

$$\frac{\bar{d}}{d\tau}I=\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\frac{\partial\lambda}{\partial\tau}\xi_0\right)+\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\frac{\partial\lambda}{\partial q_s}\xi_s\right)+\frac{\bar{d}}{d\tau}\left(\frac{1}{\lambda}\cdot\frac{\partial\lambda}{\partial p_s}\eta_s\right)+\frac{\partial\dot{q}_s}{\partial q_s}\frac{\bar{d}}{d\tau}\xi_0+\frac{\partial\dot{p}_s}{\partial p_s}\frac{\bar{d}}{d\tau}\xi_0+\frac{\partial^2\dot{q}_s}{\partial q_s\partial\tau}\xi_0+\frac{\partial^2\dot{q}_s}{\partial q_s\partial q_k}\xi_s+\frac{\partial^2\dot{q}_s}{\partial q_s\partial p_k}\eta_s+\frac{\partial^2\dot{p}_s}{\partial p_s\partial\tau}\xi_0+\frac{\partial^2\dot{p}_s}{\partial p_s\partial q_k}\xi_s+\frac{\partial^2\dot{p}_s}{\partial p_s\partial p_k}\eta_s.\quad(16)$$

Using (4),(12),(15) and (16), then have

$$\frac{\bar{d}}{d\tau}I=\frac{\partial\ln\lambda}{\partial p_s}\left[\frac{\bar{d}}{d\tau}\eta_s-\dot{p}_s\frac{\bar{d}}{d\tau}\xi_0-X^{(0)}(\dot{p}_s)\right]+\frac{\partial\ln\lambda}{\partial q_s}\left[\frac{\bar{d}}{d\tau}\xi_s-\dot{q}_s\frac{\bar{d}}{d\tau}\xi_0-X^{(0)}(\dot{q}_s)\right]=\frac{\partial\ln\lambda}{\partial p_s}\left[\frac{\bar{d}}{d\tau}\eta_s+\left(\frac{\partial H}{\partial q_s}+\frac{1-\alpha}{t-\tau}p_s\right)\frac{\bar{d}}{d\tau}\xi_0+X^{(0)}\left(\frac{\partial H}{\partial q_s}+\frac{1-\alpha}{t-\tau}p_s\right)\right]+\frac{\partial\ln\lambda}{\partial q_s}\left[\frac{\bar{d}}{d\tau}\xi_s-\frac{\partial H}{\partial p_s}\frac{\bar{d}}{d\tau}\xi_0-X^{(0)}\left(\frac{\partial H}{\partial p_s}\right)\right].\quad(17)$$

Considering equation(11), then get

$$\bar{d}I/d\tau=0.\quad(18)$$

Therefore, the theorem is proved. When ξ_0 equals to zero, the system possess the Hojman conserved quantity, such that

$$I=\frac{1}{\lambda}\frac{\partial}{\partial q_s}(\lambda\xi_s)+\frac{1}{\lambda}\frac{\partial}{\partial p_s}(\lambda\eta_s)=\text{const}.\quad(19)$$

3　The Lie Symmetry and the Noether Conserved Quantity

The existence condition and the form of Noether conserved quantity of the Lie symmetry under El-Nabulsi model in phase space will be given in the following theorem.

Theorem 2 For non-conservative dynamical system (4) under El-Nabulsi model in phase space, if the infinitesimal generators ξ_0,ξ_s,η_s satisfy the determining equations (11), and there exists with a gauge function $G=G(\tau,q_s,p_s)$ satisfying the following structure equation, such that

$$-\frac{\partial H}{\partial\tau}\xi_0-\frac{\partial H}{\partial q_s}\xi_s+\left(\dot{q}_s-\frac{\partial H}{\partial p_s}\right)\eta_s+p_s(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)+$$

$$(p_s\dot{q}_s - H)\left(\dot{\xi}_0 + \frac{1-\alpha}{t-\tau}\xi_0\right) + \dot{G}(t-\tau)^{1-\alpha} = 0 . \quad (20)$$

Then, the Lie symmetry of the system leads to the Noether conserved quantity, that is

$$I_N = (p_s\xi_s - H\xi_0)(t-\tau)^{\alpha-1} + G = \text{const.} \quad (21)$$

Proof $\frac{\mathrm{d}I_N}{\mathrm{d}\tau} = \frac{\mathrm{d}}{\mathrm{d}\tau}(p_s\xi_s - H\xi_0)(t-\tau)^{\alpha-1} + (\alpha-1)(t-\tau)^{\alpha-2}(-1)(p_s\xi_s - H\xi_0) + \dot{G} = [\dot{p}_s\xi_s + p_s\dot{\xi}_s - \dot{H}\xi_0 - H\dot{\xi}_0](t-\tau)^{\alpha-1} - (\alpha-1)(t-\tau)^{\alpha-2}(p_s\xi_s - H\xi_0) + (t-\tau)^{\alpha-1}\left[\frac{\partial H}{\partial\tau}\xi_0 + \frac{\partial H}{\partial q_s}\xi_s - (\dot{q}_s - \frac{\partial H}{\partial p_s})\eta_s - p_s(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0) - (p_s\dot{q}_s - H)(\dot{\xi}_0 + \frac{1-\alpha}{t-\tau}\xi_0)\right] = \frac{\partial H}{\partial q_s}(\xi_s - \dot{q}_s\xi_0)(t-\tau)^{\alpha-1} + \dot{p}_s(\xi_s - \frac{\partial H}{\partial p_s}\xi_0)(t-\tau)^{\alpha-1} + \frac{1-\alpha}{t-\tau}p_s(\xi_s - \dot{q}_s\xi_0)(t-\tau)^{\alpha-1} = 0 .$

4 Examples

Example 1 Let us study the extremum problem of integral functional

$$S = \frac{1}{\Gamma(\alpha)}\int_{\tau_1}^{\tau_2}\left\{p_1\dot{q}_1 + p_2\dot{q}_2 - \left[\frac{1}{2}(p_1^2+p_2^2) - \mu(q_1^2+q_2^2)^{-1/2}\right]\right\}(t-\tau)^{\alpha-1}\mathrm{d}\tau \quad (22)$$

with the fixed boundary conditions $q_s(\tau_1) = q_{s,1}$, $q_s(\tau_2) = q_{s,2}(s = 1,2,\cdots,n)$.

Problem (22) is a fractional variational problem, where

$$p_1 = \dot{q}_1, p_2 = \dot{q}_2,$$

$$H = \frac{1}{2}(p_1^2+p_2^2) - \mu(q_1^2+q_2^2)^{-1/2}. \quad (23)$$

From formulae (4), we can obtain

$\dot{q}_1 = \partial H/\partial p_1 = p_1, \dot{p}_1 = -\partial H/\partial q_1 - (1-\alpha)/(t-\tau)p_1 =$
$-\mu q_1(q_1^2+q_2^2)^{-3/2} - (1-\alpha)/(t-\tau)p_1,$
$\dot{q}_2 = \partial H/\partial p_2 = p_2,$
$\dot{p}_2 = -\partial H/\partial q_2 - (1-\alpha)/(t-\tau)p_2 =$
$-\mu q_2(q_1^2+q_2^2)^{-3/2} - (1-\alpha)/(t-\tau)p_2. \quad (24)$

From the determining equations (11), we have

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_1 - p_1\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0 = \eta_1, \frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_2 - p_2\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0 = \eta_2,$$

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\eta_1 + \left[\mu q_1(q_1^2+q_2^2)^{-3/2} + \frac{1-\alpha}{t-\tau}p_1\right]\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0 = -\frac{1-\alpha}{(t-\tau)^2}p_1\xi_0 + \left[-\mu(q_1^2+q_2^2)^{-3/2} + 3\mu q_1^2(q_1^2+q_2^2)^{-5/2}\right]\xi_1 + 3\mu q_1q_2(q_1^2+q_2^2)^{-5/2}\xi_2 - \frac{1-\alpha}{t-\tau}\eta_1,$$

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\eta_2 + \left[\mu q_2(q_1^2+q_2^2)^{-3/2} + \frac{1-\alpha}{t-\tau}p_2\right]\frac{\bar{\mathrm{d}}}{\mathrm{d}\tau}\xi_0 = -\frac{1-\alpha}{(t-\tau)^2}p_2\xi_0 + 3\mu q_1q_2(q_1^2+q_2^2)^{-5/2}\xi_1 + \left[-\mu(q_1^2+q_2^2)^{-3/2} + 3\mu q_2^2(q_1^2+q_2^2)^{-5/2}\right]\xi_2 - \frac{1-\alpha}{t-\tau}\eta_2. \quad (25)$$

The equations (25) have a solution

$$\xi_0 = 0, \xi_1 = -q_2, \xi_2 = q_1, \eta_1 = -p_2, \eta_2 = p_1. \quad (26)$$

From formula (12), then have

$$\bar{\mathrm{d}}\ln\lambda/\mathrm{d}\tau = 2(1-\alpha)/(t-\tau), \quad (27)$$

Equation(27) has a solution

$$\lambda = (q_1p_2 - p_1q_2)(t-\tau)^{3(\alpha-1)}, \quad (28)$$

Substituting the formulae (26) and (28) into (13), then obtain

$$I_1 = 0 . \quad (29)$$

The conserved quantity (29) is trivial.

When $\alpha = 1$, there is another solution to eqation (27), namely

$$\lambda = \mu q_2(q_1^2+q_2^2)^{-1/2} - q_2p_1^2 + q_1p_1p_2. \quad (30)$$

Substituting the formulae (26) and (30) into (13), then have

$$I_2 = \frac{\mu q_1(q_1^2+q_2^2)^{-1/2} - q_1p_2^2 + q_2p_1p_2}{\mu q_2(q_1^2+q_2^2)^{-1/2} - q_2p_1^2 + q_1p_1p_2}. \quad (31)$$

The structure equation (20) gives

$$-\mu q_1(q_1^2+q_2^2)^{-3/2}\xi_1 - \mu q_2(q_1^2+q_2^2)^{-3/2}\xi_2 + p_1(\dot{\xi}_1 - \dot{q}_1\dot{\xi}_0) + p_2(\dot{\xi}_2 - \dot{q}_2\dot{\xi}_0) + (p_1\dot{q}_1 + p_2\dot{q}_2 - H)\cdot(\dot{\xi}_0 + \frac{1-\alpha}{t-\tau}\xi_0) = -\dot{G}(t-\tau)^{1-\alpha}. \quad (32)$$

From formula (26) and equation (32), then can get

$$G = 0, \quad (33)$$

Substituting the formulae (26) and (33) into (21), we can obtain the Noether conserved quantity of the system, namely

$$I_{N1} = (q_1p_2 - p_1q_2)(t-\tau)^{\alpha-1} . \quad (34)$$

Example 2 Let us study the extremum problem of integral functional

$$S = \frac{1}{\Gamma(\alpha)}\int_{\tau_1}^{\tau_2}\left\{p_1\dot{q}_1 + p_2\dot{q}_2 - \left[\frac{1}{2}(p_1^2+p_2^2)\right]\right\}\cdot(t-\tau)^{\alpha-1}\mathrm{d}\tau, \quad (35)$$

with the fixed boundary conditions

$$\delta q_s|_{\tau=\tau_1} = \delta q_s|_{\tau=\tau_2} = 0(s = 1,2,\cdots,n) .$$

Problem (35) is a fractional variational problem, where

$$p_1 = \dot{q}_1, p_2 = \dot{q}_2, H = (p_1^2 + p_2^2)/2. \quad (36)$$

From formulae (4), we obtain

$$\dot{q}_1 = p_1, \dot{q}_2 = p_2, \dot{p}_1 = -\frac{1-\alpha}{t-\tau}p_1, \dot{p}_2 = -\frac{1-\alpha}{t-\tau}p_2 .$$

From the determining equations (11), then get

$$\xi_0 = 1, \xi_1 = 1, \xi_2 = 1, \eta_1 = 0, \eta_2 = 0. \quad (38)$$

From formula (12), then have

$$\lambda = (q_1 p_2 - p_1 q_2)(t-\tau)^{3(\alpha-1)}. \quad (39)$$

Substituting the formulae (38) and (39) into (13), then obtain a generalized Hojman conserved quantity of the system, namely

$$I_3 = -\frac{3(\alpha-1)}{t-\tau} + \frac{p_2 - p_1}{q_1 p_2 - p_1 q_2} = \text{const.}$$

Fromformula (38) and the structure equation (20), then have

$$G = -\frac{1}{2}\int_{\tau_1}^{\tau_2}(1-\alpha)(p_1^2 + p_2^2)(t-\tau)^{\alpha-2}\mathrm{d}\tau. \quad (41)$$

Substituting the formulae (38) and (41) into (21), then can obtain the Noether conserved quantity of the system, namely

$$I_{N2} = \left[p_1 + p_2 - (p_1^2 + p_2^2)/2\right](t-\tau)^{\alpha-1} - \int_{\tau_1}^{\tau_2}(1-\alpha)(p_1^2 + p_2^2)(t-\tau)^{\alpha-2}\mathrm{d}\tau/2 .$$

When $\alpha = 1$, the conserved quantities above become the classical conserved quantities as follows

$$I'_3 = (p_2 - p_1)/(q_1 p_2 - p_1 q_2) = \text{const},$$

$$I'_{N2} = p_1 + p_2 - (p_1^2 + p_2^2)/2 .$$

5 Conclusion

In the paper, the Lie symmetry and conserved quantity of non-conservative dynamical based on El-Nabulsi models in phase space are studied. The determining equations of the Lie symmetry based on EI-Nabulsi models in phase space are given. The generalized Hojman conserved quantity and the Noether conserved quantity are also obtained. The novelty of EI-Nabulsi model is that the fractional integral about time only needs one real parameter, and the derived equations of motion established under El-Nabulsi's dynamics model are similar to the classical ones, and doesn't contain the fractional derivatives. The study of the Lie symmetry theory based on EI-Nabulsi models can solve some problems which can't solve in the classical sense, so the results of this paper are of universal significance. Moreover, we can further study the Mei symmetry and the conserved quantity under the El-Nabulsi models and so on.

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相空间中基于 El-Nabulsi 模型的 Lie 对称性与守恒量

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摘要: 研究相空间中基于 El-Nabulsi 非保守动力学模型的 Lie 对称性与守恒量. 首先,建立系统的运动方程. 其次,在一般无限小变换下,建立确定方程,从而给出相空间中基于 El-Nabulsi 模型的 Lie 对称性的定义和判据,同时,给出相空间中 Lie 对称性直接导致的广义 Hojman 守恒量,Hojman 守恒量为广义 Hojman 守恒量一特例. 然后,给出基于 El-Nabulsi 模型的 Lie 对称性导致的 Noether 守恒量. 最后,给出 2 个特例说明结果的应用.

关键词: El-Nabulsi 模型; 相空间; Lie 对称性; 广义 Hojman 守恒量; Noether 守恒量

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