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多线性平方函数的加权 Morrey 型估计

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摘要:利用 2 进分解技术研究了一类多线性平方函数的连续性,建立了多线性平方函数在加权 Morrey 空间上的有界性,即当所有 $p_i > 1$ 时,$L^{p_1,\kappa}(\omega_1)\times\cdots\times L^{p_m,\kappa}(\omega_m)\to L^{p,\kappa}(v_{\vec{\omega}})$,当某个 $p_i = 1$ 时,$L^{p_1,\kappa}(\omega_1)\times\cdots\times L^{p_m,\kappa}(\omega_m)\to WL^{p,\kappa}(v_{\vec{\omega}})$.

关键词:多线性平方函数;权;加权 Morrey 空间

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0 引言

R. R. Coifman 等[1-2]引进并且研究了双线性 Calderón-Zygmund 算子,L. Grafakos 等[3-4]研究了多线性 Calderón-Zygmund 算子. 由于 Calderón-Zygmund 算子和 Littlewood-Paley 算子有紧密联系并且多线性 Littlewood-Paley-g 函数和相应的多线性 Littlewood-Paley 型估计在 PDE 和其它领域中有应用[5-9],如文献[9]研究了一类多线性平方函数并将其应用到非常著名的 Kato 问题. 因此,对于多线性平方函数的研究具有非常重要的意义.

首先给出多线性平方函数的定义:

$$T(\vec{f})(x)=\left(\int_0^\infty\left|\int_{(\mathbf{R}^n)^m}K_t(x,y_1,\cdots,y_m)\cdot\prod_{i=1}^m f_i(y_i)\,\mathrm{d}y_1\cdots\mathrm{d}y_m\right|^2\frac{\mathrm{d}t}{t}\right)^{1/2}.\quad(1)$$

假设 T 是 $L^{q_1}\times\cdots\times L^{q_m}\to L^q$ 且有界,其中 $1\leqslant q_i<\infty$,$1/q=1/q_1+\cdots+1/q_m$. $\forall t\in(0,\infty)$,设 $K_t(x,y_1,\cdots,y_m)$ 是一个在 $(\mathbf{R}^n)^{m+1}$ 上远离对角线 $x=y_1=\cdots=y_m$ 的局部可积函数,记 $(x,\vec{y})=(x,y_1,\cdots,y_m)$.

若对某个 $\gamma>0$,A 及 $B>1$,则有

$$\left(\int_0^\infty|K_t(x,\vec{y})|^2\frac{\mathrm{d}t}{t}\right)^{1/2}\leqslant A/\left(\sum_{i=1}^m|x-y_i|\right)^{mn},\quad(2)$$

当 $|z-x|\leqslant\max_i|x-y_i|/B$,$i\in1,2,\cdots,m$ 时,有

$$\left(\int_0^\infty\left|K_t(z,\vec{y})-K_t(x,\vec{y})\right|^2\frac{\mathrm{d}t}{t}\right)^{1/2}\leqslant A|z-x|^\gamma/\left(\sum_{i=1}^m|x-y_i|\right)^{mn+\gamma},\quad(3)$$

当 $|y-y_i'|\leqslant|x-y_i|/B$,$i\in1,2,\cdots,m$ 时,有

$$\left(\int_0^\infty\left|K_t(x,y_1,\cdots,y_i,\cdots y_m)-K_t(x,y_1,\cdots,y_i',\cdots,y_m)\right|^2\frac{\mathrm{d}t}{t}\right)^{1/2}\leqslant A|y_i-y_i'|^\gamma/\left(\sum_{i=1}^m|x-y_i|\right)^{mn+\gamma}.\quad(4)$$

进一步地,文献[10]考虑了多线性 Calderón-Zygmund 算子 T 的 $\prod_{i=1}^m L^{p_i,\kappa}(\omega_i)\to L^p(v_{\vec{\omega}})$ 上的有界性;文献[11]引进了多线性平方函数 T,并且考虑了 $\prod_{i=1}^m L^{p_i}(\omega_i)\to L^p(\omega)$ 的有界性. 受此启发,本文考虑了多线性平方函数在加权 Morrey 空间上的有界性. 对加权 Morrey 空间上的算子及其交换子的有界性的研究参见文献[12-15].

1 基本概念和主要结论

定义1 设 $1\leqslant p_1,\cdots,p_m<\infty$,$p$ 满足 $1/p=1/p_1+\cdots+1/p_m$,给定 $\vec{\omega}=(\omega_1,\cdots,\omega_m)$,设 $v_{\vec{\omega}}=\prod_{i=1}^m\omega_i^{p/p_i}$. 若 $\sup_Q\left(\frac{1}{|Q|}\int_Q\prod_{i=1}^m\omega_i^{p/p_i}\right)^{1/p}\prod_{i=1}^m\left(\frac{1}{|Q|}\int_Q\omega_i^{1-p_i'}\right)^{1/p_i'}<\infty$,则称 $\vec{\omega}\in A_{\vec{P}}$. 当 $\vec{P}=(1,\cdots,1)$ 时,$\left(\int_Q\omega_i^{1-p_i'}/|Q|\right)^{1/p_i'}$ 理解为 $(\inf_Q\omega_i)^{-1}$.

定义2 设 $1\leqslant p<\infty$,$0<\kappa<1$,ω 是 $\mathbf{R}^n$ 上的权函数. 加权 Morrey 空间为

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$$L^{p,\kappa}(\omega) = \{f \in L^p_{loc}: \|f\|_{L^{p,\kappa}(\omega)} < \infty\},$$

其中 $\|f\|_{L^{p,\kappa}(\omega)} = \sup\limits_{Q}\left(\frac{1}{\omega(Q)^{\kappa}}\int_Q |f(x)|^p\omega(x)\,dx\right)^{1/p}$.

加权弱 Morrey 空间为

$$WL^{p,\kappa}(\omega) = \{f \text{ 可测}: \|f\|_{WL^{p,\kappa}(\omega)} < \infty\},$$

其中 $\|f\|_{WL^{p,\kappa}(\omega)} = \sup\limits_{Q}\sup\limits_{\lambda>0}\frac{\lambda}{\omega(Q)^{\kappa/p}}\omega(\{x \in Q: |f(x)| > \lambda\})^{1/p}$.

若存在常数 $C > 0$，使得对任意的方体 Q 有 $\omega(2Q) \leqslant C\omega(Q)$，则称权 ω 满足双倍条件.

下面给出一个关键引理.

引理1[11] 设 T 是(1)式定义的多线性平方函数，核满足条件(2)~(4). 设 $1/p = 1/p_1 + \cdots + 1/p_m$，$1 \leqslant p_i < \infty$，$\vec{\omega}$ 满足 $A_{\vec{P}}$ 权条件，则

(i) 当所有 $p_i > 1$，存在常数 C 使得

$$\|T(\vec{f})\|_{L^p(v_{\vec{\omega}})} \leqslant C\prod_{i=1}^m \|f_i\|_{L^{p_i}(\omega_i)};$$

(ii) 当某个 $p_i = 1$，存在常数 C 使得

$$\|T(\vec{f})\|_{L^{p,\infty}(v_{\vec{\omega}})} \leqslant C\prod_{i=1}^m \|f_i\|_{L^{p_i}(\omega_i)}.$$

最后给出本文的主要结论.

定理1 设 T 是(1)式定义的多线性平方函数，核满足条件(2)~(4). 设 $1/p = 1/p_1 + \cdots + 1/p_m$，其中 $1/m \leqslant p < \infty$，$1 \leqslant p_i \leqslant \infty$ 且 $0 < \kappa < 1$，$\omega_i \in A_{p_i}$，$v_{\vec{\omega}} = \sum\limits_{i=1}^m \omega_i^{p/p_i}$，则

(i) 当所有 $p_i > 1$ 时，存在常数 C 使得

$$\|T(\vec{f})\|_{L^{p,\kappa}(v_{\vec{\omega}})} \leqslant C\prod_{i=1}^m \|f_i\|_{L^{p_i,\kappa}(\omega_i)};$$

(ii) 当某个 $p_i = 1$ 时，存在常数 C 使得

$$\|T(\vec{f})\|_{WL^{p,\kappa}(v_{\vec{\omega}})} \leqslant C\prod_{i=1}^m \|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

2 定理的证明

事实上，对任意的方体 $Q \in \mathbf{R}^n$，分解 $f_i = f_i^0 + f_i^\infty$，其中 $f_i^0 = f_i\chi_{Q^*}$，$i = 1,\cdots,m$ 和 $Q^* = 8Q$，则

$$\prod_{i=1}^m f_i(y_i) = \prod_{i=1}^m (f_i^0(y_i) + f_i^\infty(y_i)) = \sum_{\alpha_1,\cdots,\alpha_m \in \{0,\infty\}} \prod_{i=1}^m f_i^{\alpha_i}(y_i) = \prod_{i=1}^m f_i^0(y_i) + \sum{}' f_1^{\alpha_1}(y_1)\cdots f_m^{\alpha_m}(y_m),$$

其中 $\sum{}'$ 中的每一部分包含至少 1 个 $\alpha_i \neq 0$，从而

$$\frac{1}{v_{\vec{\omega}}(Q)^{\kappa/p}}\left(\int_Q |T(f_1,\cdots,f_m)(x)|^p v_{\vec{\omega}}(x)\,dx\right)^{1/p} \leqslant$$

$$\frac{1}{v_{\vec{\omega}}(Q)^{\kappa/p}}\left(\int_Q |T(f_1^0,\cdots,f_m^0)(x)|^p v_{\vec{\omega}}(x)\,dx\right)^{1/p} + \sum{}'\frac{1}{v_{\vec{\omega}}(Q)^{\kappa/p}}\left(\int_Q |T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|^p v_{\vec{\omega}}(x)\,dx\right)^{1/p} = I^{0,\cdots,0} + \sum{}' I^{\alpha_1,\cdots,\alpha_m}.$$

由引理1(i)，定义2和双倍条件，有

$$I^{0,\cdots,0} \leqslant \frac{1}{v_{\vec{\omega}}(Q)^{\kappa/p}}\left(\int_{\mathbf{R}^n} |T(f_1^0,\cdots,f_m^0)(x)|^p v_{\vec{\omega}}(x)\,dx\right)^{1/p} \leqslant \frac{C}{v_{\vec{\omega}}(Q)^{\kappa/p}}\prod_{i=1}^m\left(\int_{\mathbf{R}^n}|f_i^0(x)|^{p_i}\omega_i(x)\,dx\right)^{1/p_i} \leqslant$$

$$C\frac{\prod\limits_{i=1}^m \omega_i(Q^*)^{\kappa/p_i}}{v_{\vec{\omega}}(Q)^{\kappa/p}}\prod_{i=1}^m \|f_i\|_{L^{p_i,\kappa}(\omega_i)} \leqslant C\prod_{i=1}^m \|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

接下来估计 $\sum{}' I^{\alpha_1,\cdots,\alpha_m}$，首先估计 $\alpha_1 = \cdots = \alpha_m = \infty$ $\forall x,z \in Q$，使得 $|x - z| < l_Q$，利用 Minkowski 不等式，核条件(1)，(2)，Hólder 不等式和 $\omega_i \in A_{p_i}$，有

$$|T(f_1^\infty,\cdots,f_m^\infty)(x)| = \left(\int_0^\infty\left|\int_{(\mathbf{R}^n)^m} K_t(x,\vec{y})f_1^\infty(y_1),\cdots,f_m^\infty(y_m)\,dy_1\cdots dy_m\right|^2\frac{dt}{t}\right)^{1/2} \leqslant$$

$$\int_{(\mathbf{R}^n\backslash Q^*)^m}\left(\int_0^\infty\left|K_t(x,\vec{y}) - K_t(z,\vec{y})\right|^2\frac{dt}{t}\right)^{1/2}\prod_{i=1}^m|f_i(y_i)|\,d\vec{y} + \int_{(\mathbf{R}^n\backslash Q^*)^m}\left(\int_0^\infty|K_t(z,\vec{y})|^2\frac{dt}{t}\right)^{1/2}\prod_{i=1}^m|f_i(y_i)|\,d\vec{y} \leqslant$$

$$\sum_{l=1}^\infty\int_{(8^{l+1}Q\backslash 8^lQ)^m}\frac{A|x - z|^\gamma}{(|z - y_i|)^{mn+\gamma}}\prod_{i=1}^m|f_i(y_i)|\,d\vec{y} + \sum_{l=1}^\infty\int_{(8^{l+1}Q\backslash 8^lQ)^m}\frac{A}{(|z - y_i|)^{mn}}\prod_{i=1}^m|f_i(y_i)|\,d\vec{y} \leqslant$$

$$C\sum_{l=1}^\infty\prod_{i=1}^m\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i \leqslant$$

$$C\sum_{l=1}^\infty\prod_{i=1}^m\frac{1}{|8^{l+1}Q|}\left(\int_{8^{l+1}Q}|f_i(y_i)|^{p_i}\omega_i(y_i)\,dy_i\right)^{1/p_i}\cdot\left(\int_{8^{l+1}Q}\omega_i(y_i)^{1-p_i'}\,dy_i\right)^{1/p_i'} \leqslant C\sum_{l=1}^\infty\prod_{i=1}^m\frac{\omega_i(8^{l+1}Q)^{\kappa/p_i}}{|8^{l+1}Q|}\cdot\|f_i\|_{L^{p_i,\kappa}(\omega_i)}\frac{|8^{l+1}Q|}{\omega_i(8^{l+1}Q)^{1/p_i}} \leqslant$$

$$C\sum_{l=1}^\infty v_{\vec{\omega}}(8^{l+1}Q)^{(\kappa-1)/p}\prod_{i=1}^m\|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

因为 $v_{\vec{\omega}} \in A_{mp}$，则 $\exists\delta > 0$ 使得 $v_{\vec{\omega}}(Q)/v_{\vec{\omega}}(8^{l+1}Q) \leqslant C(Q/|8^{l+1}Q|)^\delta$. 因此

$$I^{\infty,\cdots,\infty} \leqslant C\sum_{l=1}^\infty\left(\frac{v_{\vec{\omega}}(Q)}{v_{\vec{\omega}}(8^{l+1}Q)}\right)^{(1-\kappa)/p}\prod_{i=1}^m\|f_i\|_{L^{p_i,\kappa}(\omega_i)} \leqslant$$

$$C\sum_{l=1}^\infty\left(\frac{|Q|}{|8^{l+1}Q|}\right)^{\delta(1-\kappa)/p}\prod_{i=1}^m\|f_i\|_{L^{p_i,\kappa}(\omega_i)} \leqslant$$

$C\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}$.

其次估计 $\alpha_{i_1}=\cdots=\alpha_{i_j}=0,\{\alpha_{i_1},\cdots,\alpha_{i_j}\}\subset\{1,\cdots,m\}$,$1\leqslant j<m$,类似 $I^{\infty,\cdots,\infty}$ 的估计,有

$$|T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|=$$

$$\left(\int_0^\infty\left|\int_{(\mathbf{R}^n)^m}K_t(x,\vec{y})\prod_{i=1}^{m}f_i^{\alpha}(y_i)\,d\vec{y}\right|^2\frac{dt}{t}\right)^{1/2}\leqslant$$

$$\int_{(\mathbf{R}^n)^m}\left(\int_0^\infty|K_t(x,\vec{y})-K_t(z,\vec{y})|^2\frac{dt}{t}\right)^{1/2}\cdot$$

$$\prod_{i=1}^{m}|f_i^{\alpha_i}(y_i)|\,d\vec{y}+\int_{(\mathbf{R}^n)^m}\left(\int_0^\infty|K_t(z,\vec{y})|^2\frac{dt}{t}\right)^{1/2}\cdot$$

$$\prod_{i=1}^{m}|f_i^{\alpha_i}(y_i)|\,d\vec{y}\leqslant C\prod_{i\in\{i_1,\cdots,i_j\}}\int_{Q^*}|f_i(y_i)|\,dy_i\cdot$$

$$\Big[\int_{(\mathbf{R}^n\backslash Q^*)^{m-j}}A\,|x-z|^\gamma\prod_{i\notin\{i_1,\cdots,i_j\}}|f_i(y_i)|\,dy_i/$$

$$\sum_{i\notin\{i_1,\cdots,i_j\}}(|z-y_i|)^{mn+\gamma}+\int_{(\mathbf{R}^n\backslash Q^*)^{m-j}}A\prod_{i\notin\{i_1,\cdots,i_j\}}|f_i(y_i)|\,dy_i/$$

$$\sum_{i\notin\{i_1,\cdots,i_j\}}(|z-y_i|)^{mn}\Big]\leqslant C\prod_{i\in\{i_1,\cdots,i_j\}}\int_{Q^*}|f_i(y_i)|\,dy_i\cdot$$

$$\sum_{l=1}^{\infty}\prod_{i\notin\{i_1,\cdots,i_j\}}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q\backslash 8^lQ}|f_i(y_i)|\,dy_i\leqslant$$

$$C\sum_{l=1}^{\infty}\prod_{i=1}^{m}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i\leqslant$$

$$C\sum_{l=1}^{\infty}\upsilon_{\vec{\omega}}(8^{l+1}Q)^{(\kappa-1)/p}\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

类似 $I^{0,\cdots,0}$ 的估计方法,得

$$I^{0,\cdots,0}\leqslant\frac{1}{\upsilon_{\vec{\omega}}(Q)^{\kappa/p}}\left(\int_Q|T(f_1^0,\cdots,f_m^0)(x)|^p\cdot\right.$$

$$\left.\upsilon_{\vec{\omega}}(x)\,dx\right)^{1/p}\leqslant C\sum_{l=1}^{\infty}\left(\frac{\upsilon_{\vec{\omega}}(Q)}{\upsilon_{\vec{\omega}}(8^{l+1}Q)}\right)^{(1-\kappa)/p}\leqslant$$

$$C\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)},$$

综上所述,结合 $I^{0,\cdots,0}$ 和 $I^{\alpha_1,\cdots,\alpha_m}$ 的估计,对 $\mathbf{R}^n$ 上的所有方体 Q 取上确界,得到结论.

(ii) 为了完成弱型估计,$\forall\lambda>0$,记

$$\upsilon_{\vec{\omega}}(\{x\in Q:|T(f_1,\cdots,f_m)(x)|>\lambda\})^{1/p}\leqslant$$

$$\upsilon_{\vec{\omega}}(\{x\in Q:|T(f_1^0,\cdots,f_m^0)(x)|>\lambda\})^{1/p}+$$

$$\sum{}'\upsilon_{\vec{\omega}}(\{x\in Q:|T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|>\lambda\})^{1/p}=$$

$$II^{0,\cdots,0}+\sum{}'II^{\alpha_1,\cdots,\alpha_m}.$$

由引理 1(ii) 及定义 2,有

$$II^{0,\cdots,0}\leqslant\frac{C}{\lambda}\prod_{i=1}^{m}\left(\int_{\mathbf{R}^n}|f_i^0(x)|^{p_i}\omega_i(x)\,dx\right)^{1/p_i}\leqslant$$

$$\frac{C\upsilon_{\vec{\omega}}(Q)^{\kappa/p}}{\lambda}\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

接着来估计 $II^{\alpha_1,\cdots,\alpha_m}$,由(i) 的证明过程知,

$$|T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|\leqslant$$

$$C\sum_{l=1}^{\infty}\prod_{i=1}^{m}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i.$$

从而,至少有 1 个 $p_i=1$,设 $\{i_1,\cdots,i_j\}\subset\{1,\cdots,m\}$,使得 $p_{i_1}=\cdots=p_{i_j}=1$,其他项大于 1,利用 Hölder 不等式和 $\omega_i\in A_{p_i}$,有

$$|T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|\leqslant$$

$$C\sum_{l=1}^{\infty}\prod_{i=1}^{m}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i\leqslant$$

$$C\sum_{l=1}^{\infty}\prod_{i\in\{i_1,\cdots,i_j\}}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i\cdot$$

$$\prod_{i\notin\{i_1,\cdots,i_j\}}\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,dy_i\leqslant$$

$$C\sum_{l=1}^{\infty}\prod_{i\in\{i_1,\cdots,i_j\}}\left(\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|\,\omega_i(y_i)\,dy_i\right)\cdot$$

$$\left(\inf_{y_i\in 8^{l+1}Q}\omega_i(y_i)\right)^{-1}\cdot\prod_{i\notin\{i_1,\cdots,i_j\}}\left(\frac{1}{|8^{l+1}Q|}\int_{8^{l+1}Q}|f_i(y_i)|^{p_i}\cdot\right.$$

$$\left.\omega_i(y_i)\,dy_i\right)^{1/p_i}\cdot\left(\int_{8^{l+1}Q}\omega_i(y_i)^{1-p_i'}dy_i\right)^{1/p_i'}\leqslant$$

$$C\prod_{i\in\{i_1,\cdots,i_j\}}\frac{1}{\omega_i(8^{l+1}Q)^{(1-\kappa)/p_i}}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}\cdot$$

$$\prod_{i\notin\{i_1,\cdots,i_j\}}\frac{1}{\omega_i(8^{l+1}Q)^{(1-\kappa)/p_i}}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}\leqslant$$

$$\frac{C}{\upsilon_{\vec{\omega}}(8^{l+1}Q)^{(1-\kappa)/p}}\cdot\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}.$$

设 $\{x\in Q:|T(f_1^{\alpha_1},\cdots,f_m^{\alpha_m})(x)|>\lambda\}\neq 0$,则 $\upsilon_{\vec{\omega}}(Q)^{1/p}\leqslant C\upsilon_{\vec{\omega}}(Q)^{\kappa/p}\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}/\lambda$. 故

$$II^{\alpha_1,\cdots,\alpha_m}\leqslant\upsilon_{\vec{\omega}}(Q)^{1/p}\leqslant C\upsilon_{\vec{\omega}}(Q)^{\kappa/p}\prod_{i=1}^{m}\|f_i\|_{L^{p_i,\kappa}(\omega_i)}/\lambda.$$

对 $\mathbf{R}^n$ 上的所有方体 Q,$\lambda>0$ 取上确界,则定理 1 证毕.

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The Weighted Morrey Type Estimates for Multilinear Square Function

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Abstract: The continuity of a class of multilinear square function is studied and a binary decomposition technique is used to obtain the boundness of multilinear square function T on weighted Morrey spaces. That is,the $L^{p_1,\kappa}(\omega_1)\times\cdots\times L^{p_m,\kappa}(\omega_m)\to L^{p,\kappa}(\upsilon_{\vec{\omega}})$ estimate of T on each $p_i>1$ and weak type $L^{p_1,\kappa}(\omega_1)\times\cdots\times L^{p_m,\kappa}(\omega_m)\to WL^{p,\kappa}(\upsilon_{\vec{\omega}})$ estimate of T when there is some $p_i=1$ are established.

Key words: multilinear square function; weight; weighted Morrey spaces

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