第 43 卷 第 1 期　江西师范大学学报(自然科学版)　Vol. 43 No. 1
2019 年 1 月　Journal of Jiangxi Normal University(Natural Science)　Jan. 2019

文章编号:1000-5862(2019)01-0035-04

5 次非线性 Schrödinger 方程的一个线性化 4 层紧致差分格式

翟步祥[1],聂　涛[1*],薛　翔[2]

(1. 南京科技职业学院基础科学学部,江苏 南京　210048; 2. 南京信息工程大学数学与统计学院,江苏 南京　210044)

摘要: 对 5 次非线性 Schrödinger 方程提出了一个线性化 4 层紧致有限差分格式,引入"抬升"技巧,运用标准的能量方法和数学归纳法建立了误差的最优估计,证明数值解在空间和时间 2 个方向分别具有 4 阶和 2 阶精度. 数值实验对理论结果进行了验证,并通过对比表明该文格式在保持精度相当的前提下较已有格式具有更高的计算效率.

关键词: 5 次非线性 Schrödinger 方程; 紧致有限差分格式; 最优误差估计; 线性化 4 层格式; 计算效率

中图分类号: O 241.8　**文献标志码:** A　**DOI:** 10.16357/j.cnki.issn1000-5862.2019.01.07

0　引言

非线性 Schrödinger 方程在量子物理、等离子物理和非线性光学等领域中有着非常广泛应用. 本文研究如下一类 5 次非线性 Schrödinger 方程的初边值问题[1]:

$$\begin{cases} \mathrm{i}u_t + u_{xx} - (|u|^2 + |u|^4)u = \\ f(x,t)u, (x,t) \in (a,b) \times (0,T], \\ u(x,0) = u_0(x), x \in [a,b], \\ u(a,t) = u(b,t) = 0, t \in (0,T], \end{cases} \tag{1}$$

其中 $f(x,t)$ 为实值函数, $u(x,t)$ 为复值函数, $u_0(x)$ 为已知的复值函数.

近年来,已有大量文献对非线性 Schrödinger 方程进行了数值研究. Bao Weizhu 等[3-5] 运用时间分裂拟谱方法对非线性 Schrödinger/Gross-Pitaeviskii 方程进行了数值求解,并数值模拟了一些物理现象. J. Argyris 等[6-9] 利用有限元法数值求解该方程. M. Dehghan 等[10-11] 对该方程引入了无网格法. 文献[12-14] 提出用谱方法和拟谱方法对该方程进行数值近似和误差分析. 因编程简单并能保持原问题的某些守恒性质,有限差分法广泛应用于非线性 Schrödinger 方程的数值模拟[15-18]. 然而关于 5 次非线性 Schrödinger 方程的数值研究尚不多见. 张鲁明等[1-2] 用有限差分法研究了一类广泛的 5 次非线性 Schrödinger 方程,同时给出了 L^2 范数下的最优误差估计. 王询等[19] 用待定系数的方法构建了一类 5 点有限差分格式,该格式族在选取适当的参数后,其计算精度在空间可达 4 阶,然而计算中在每一个时间步都需要求解一个 5 对角代数方程组. 为提高精度,文献[20] 提出一个非线性紧致有限差分格式,数值计算中不可避免的需要迭代,从而耗费大量机时. 鉴于以上分析,本文旨在对问题(1) 构造一个线性化 4 层紧致有限差分格式,使得新格式在保持精度相当的前提下具有更高的计算效率.

1　有限差分格式

1.1　相关符号的定义

取正整数 J、K,时空方向上的步长分别为 $\tau = T/N$, $h = (b-a)/J$,网格点分别为 $t_n = n\tau(n = 0,1,\cdots,N)$, $x_j = a + jh(j = 0,1,\cdots,J)$,并记 $U_j^n = u(x_j,t_n)$, $u_j^n \approx u(x_j,t_n)$. 为书写简单,引入以下记号:

$\delta_x^+ w_j^n = (w_{j+1}^n - w_j^n)/h$, $\delta_t^+ w_j^n = (w_j^{n+1} - w_j^n)/\tau$, $\delta_x^2 w_j^n = (w_{j+1}^n - 2w_j^n + w_{j-1}^n)/h^2$, $\mathscr{A}_h w_j^n = (1 + h^2\delta_x^2/12)w_j^n = (w_{j-1}^n + 10w_j^n + w_{j+1}^n)/12$.

定义网格函数空间 $X_h = \{w \mid w = (w_0,w_1,\cdots,w_{J-1},w_J), w_0 = w_J = 0\}$. 设 u,v 为 X_h 上的任意 2 个网格函数,其内积和范数定义为 $\langle u,v\rangle = h\sum_{j=1}^{J-1} u_j\bar{v}_j$, $\|u^n\| = \left(h\sum_{j=1}^{J-1}|u_j^n|^2\right)^{1/2}$.

收稿日期:2018-05-20

基金项目:国家自然科学基金(11571181)和江苏省自然科学基金(BK20171454)资助项目.

通信作者:聂　涛(1979-),女,山东青岛人,副教授,主要从事偏微分方程数值解的研究. E-mail: nietaonjcc@126.com

本文需要用到如下引理:

引理 1[21] 对于任意一个网格函数 $u\in X_h$,有 $\|\delta_x^+u^n\|\le 4\|u^n\|/h$,$\|\delta_x^2u^n\|\le 4\|\delta_x^+u^n\|/h.$

引理 2[22] 若网格函数 $u\in X_h$,则

$C_1\|u^n\|\le\|\mathscr{A}_h^{-1}u^n\|\le C_2\|u^n\|.$

引理 3[21] 任给网格函数 $u\in X_h$,有

$\|u^n\|_\infty\le\sqrt{b-a}\|\delta_x^+u^n\|/2.$

1.2 有限差分格式

对初边值问题(1),提出如下4层紧致有限差分格式:

$$\mathrm{i}\mathscr{A}_h\delta_t^+u_j^n+\delta_x^2(u_j^{n+1}+u_j^n)/2-\mathscr{A}_h[(|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^2+|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^4)(u_j^{n+1}+u_j^n)]/2=\mathscr{A}_h(f_j^{n+1}u_j^{n+1}+f_j^nu_j^n)/2,j=1,2,\cdots,J-1,n=1,2,\cdots,N-1;\tag{2}$$

$$u_j^0=u_0(x_j),j=0,1,\cdots;\tag{3}$$

$$u_j^1=u_0(x_j)+\tau u_t(x_j,0),j=0,1,\cdots,J;\tag{4}$$

$$u_j^2=u_0(x_j)+2\tau u_t(x_j,0),j=0,1,\cdots,J;\tag{5}$$

$$u_0^n=u_J^n,n=1,2,\cdots,N;\tag{6}$$

$$u_t(x_j,0)=\mathrm{i}[\partial_{xx}u_0(x_j)-(|u_0(x_j)|^2+|u_0(x_j)|^4)u_0(x_j)-f(x_j,0)u_0(x_j)],j=1,2,\cdots,J-1,\tag{7}$$

其中 $f_j^n=f(x_j,t_n)$,在(2)式两端同时作用算子 $\mathscr{A}_h^{-1}$,则(2)式可写成如下等价形式

$$\mathrm{i}\delta_t^+u_j^n+\mathscr{A}_h^{-1}\delta_x^2(u_j^{n+1}+u_j^n)/2-[(|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^2+|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^4)(u_j^{n+1}+u_j^n)]/2=(f_j^{n+1}u_j^{n+1}+f_j^nu_j^n)/2,j=1,2,\cdots,J-1,n=1,2,\cdots,N-1.$$

2 误差估计

记格式(2)~(6)的局部截断误差为 $R_j^n(j=1,2,\cdots,J-1,n=0,1,\cdots,N-1)$,其定义如下:

$$R_j^0=u(x_j,\tau)-u_0(x_j)-\tau u_t(x_j,0),j=1,2,\cdots,J-1,$$
$$R_j^1=u(x_j,2\tau)-u_0(x_j)-2\tau u_t(x_j,0),j=1,2,\cdots,J-1,$$
$$R_j^n=\mathrm{i}\delta_t^+U_j^n+\mathscr{A}_h^{-1}\delta_x^2(U_j^{n+1}+U_j^n)/2-[(|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^2+|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^4)(U_j^{n+1}+U_j^n)]/2-(f_j^{n+1}U_j^{n+1}+f_j^nU_j^n)/2,j=1,2,\cdots,J-1,n=2,3,\cdots,N-1,\tag{8}$$

其中 $U_j^n=u(x_j,t_n)$.运用 Taylor 展开可得引理 5.

引理 5 格式(2)~(6)的局部截断误差满足

$R_j^0=O(\tau^2),R_j^1=O(\tau^2),R_j^n=O(\tau^2+h^4),j=1,2,\cdots,J,n=2,3,\cdots,N-1.$

定义误差函数 $e_j^n=U_j^n-u_j^n,j=0,1,2,\cdots,J,n=0,1,2,\cdots,N$,则有

定理 1 假设 $u\in C^{6,3}([a,b]\times[0,T])$,则差分格式(2)~(6)的解以 $\|\cdot\|$ 范数收敛到初边值问题(1)的解,收敛阶为 $O(\tau^2+h^4)$.

证 将(8)式分别与(4)式、(5)式和(7)式相减可得如下误差方程

$$e_j^1=R_j^0,j=1,2,\cdots,J-1;\tag{9}$$

$$e_j^2=R_j^1,j=1,2,\cdots,J-1;\tag{10}$$

$$\mathrm{i}\delta_t^+e_j^n+\mathscr{A}_h^{-1}\delta_x^2(e_j^{n+1}+e_j^n)/2-\omega_j^{n+1}/2-(f_j^{n+1}e_j^{n+1}+f_j^ne_j^n)/2=R_j^n,j=1,2,\cdots,J-1,n=2,3,\cdots,N-1,\tag{11}$$

其中 $\omega_j^{n+1}=(|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^2+|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^4)(U_j^n+U_j^{n+1})-(|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^2+|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^4)(u_j^n+u_j^{n+1})=(|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^2+|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^4)(e_j^n+e_j^{n+1})+\{(U_j^n+U_j^{n-1}/2-U_j^{n-2}/2)(\bar{e}_j^n+\bar{e}_j^{n-1}/2-\bar{e}_j^{n-2}/2)+(e_j^n+e_j^{n-1}/2-e_j^{n-2}/2)(\bar{u}_j^n+\bar{u}_j^{n-1}/2-\bar{u}_j^{n-2}/2)+(|U_j^n+U_j^{n-1}/2-U_j^{n-2}/2|^2+|u_j^n+u_j^{n-1}/2-u_j^{n-2}/2|^2)[(U_j^n+U_j^{n-1}/2-U_j^{n-2}/2)(\bar{e}_j^n+\bar{e}_j^{n-1}/2-\bar{e}_j^{n-2}/2)+(e_j^n+e_j^{n-1}/2-e_j^{n-2}/2)(\bar{u}_j^n+\bar{u}_j^{n-1}/2-\bar{u}_j^{n-2}/2)]\}(u_j^n+u_j^{n+1}),n=2,3,\cdots,N-1.$

由(9)~(10)式和引理 5 可得

$$\|e^1\|\le C\tau^2,\|e^2\|\le C\tau^2.\tag{12}$$

由 Taylor 展开得

$\delta_x^+R_j^l\le O(\tau^2),\delta_x^2R_j^l\le O(\tau^2),l=0,1,$

所以有

$\|\delta_x^+e^1\|=\|\delta_x^+R^0\|\le C\tau^2,\|\delta_x^2e^1\|=\|\delta_x^2R^0\|\le C\tau^2,\|e^1\|_\infty\le\|e^1\|+\|\delta_x^+e^1\|\le C\tau^2\le C(\tau^2+h^4).$

现假设当 $n\le k(1\le k<N)$ 时,有

$\|e^n\|\le C(\tau^2+h^4),\|\delta_x^2e^n\|\le C.$

进而可得

$\|u^n\|_\infty\le\|U^n\|_\infty+\|e^n\|_\infty\le C,$

$\|\omega^{n+1}\|\le C(\|e^{n-2}\|+\|e^{n-1}\|+\|e^n\|+\|e^{n+1}\|)\le C(\tau^2+h^4),n=2,3,\cdots,k.$

将(11)式与 e^n+e^{n+1} 作内积,取虚部得

$(\|e^{n+1}\|^2-\|e^n\|^2)/\tau+\mathrm{Im}\langle\mathscr{A}_n^{-1}\delta_x^2(e^n+e^{n+1})/2,(e^n+e^{n+1})\rangle-\mathrm{Im}\langle\omega^{n+1},(e^n+e^{n+1})\rangle/2-\mathrm{Im}\langle f^{n+1}e^{n+1}+f^ne^n,(e^n+e^{n+1})\rangle/2=\mathrm{Im}\langle R^n,(e^n+e^{n+1})\rangle,n=2,3,\cdots,k.$

又

$\mathrm{Im}\langle\mathscr{A}_h^{-1}\delta_x^2(e^n+e^{n+1}),(e^n+e^{n+1})\rangle=0,$

$\mathrm{Im}\langle f^{n+1}e^{n+1}+f^ne^n,(e^n+e^{n+1})\rangle\le C(\|e^n\|^2+\|e^{n+1}\|^2),$

$$\mathrm{Im}\langle R^n,(e^n+e^{n+1})\rangle \leqslant C(\|R^n\|+\|e^n\|+\|e^{n+1}\|),$$

$$|\langle \omega^{n+1},(e^n+e^{n+1})\rangle| \leqslant C(\|e^{n-2}\|^2+\|e^{n-1}\|^2+\|e^n\|^2+\|e^{n+1}\|^2), \quad (13)$$

由(13)式得

$$\|e^{n+1}\|^2-\|e^n\|^2 \leqslant C\tau(\|e^{n-2}\|^2+\|e^{n-1}\|^2+\|e^n\|^2+\|e^{n+1}\|^2)+C\tau\|R^n\|^2, n=2,3,\cdots,k.$$

上式对 n 求和，有 $\|e^{n+1}\|^2-\|e^2\|^2 \leqslant C\tau\cdot\sum_{l=1}^{n+1}\|e^l\|^2+C(\tau^2+h^4)^2$. 当 τ 足够小时，由 Gronwall 不等式和(12)式得 $\|e^{n+1}\| \leqslant C(\tau^2+h^4)$. 由(11)式可得

$$\|\mathscr{A}_h^{-1}\delta_x^2(e^n+e^{n+1})\|/2=\|-\mathrm{i}\delta_t^+e^n+\omega^{n+1}/2+(f^{n+1}e^{n+1}+f^ne^n)/2+R^n\| \leqslant C\|\delta_t^+e^n\|+C\|\omega^{n+1}\|+C\|f^{n+1}e^{n+1}+f^ne^n\|+\|R^n\| \leqslant C\tau^{-1}(\|e^{n+1}\|+\|e^n\|)+C(\tau^2+h^4) \leqslant C\tau^{-1}(\tau^2+h^4),$$

运用引理 2 有

$$\|\delta_x^2e^{n+1}\|-\|\delta_x^2e^n\| \leqslant \|\delta_x^2(e^{n+1}+e^n)\| \leqslant C\|\mathscr{A}_h^{-1}\delta_x^2(e^{n+1}+e^n)\| \leqslant C\tau^{-1}(\tau^2+h^4),$$

上式对 n 求和得

$$\|\delta_x^2e^{n+1}\| \leqslant C(n+1)\tau^{-1}(\tau^2+h^4) \leqslant C\tau^{-2}(\tau^2+h^4) \leqslant C+C\tau^{-2}h^4,$$

只要当 $h \leqslant \tau$ 时，有 $h^4 \leqslant \tau^2$，此时有 $\|\delta_x^2e^{n+1}\| \leqslant C$.

由引理 1 可得

$$\|\delta_x^2e^{n+1}\| \leqslant 16\|e^{n+1}\|/h^2 \leqslant Ch^{-2}(h^4+\tau^2) \leqslant C(h^2+\tau^2h^{-2}),$$

由此可见，当 $\tau \leqslant h$ 时，有 $\tau^2 \leqslant h^2$，从而 $\|\delta_x^2e^{n+1}\| \leqslant C$. 所以，无论网格比如何，总有

$$\|\delta_x^+e^{n+1}\| \leqslant C\|\delta_x^2e^{n+1}\| \leqslant C.$$

即当 $n=k+1$ 时，有

$$\|e^n\| \leqslant C(\tau^2+h^4), \|\delta_x^2e^n\| \leqslant C.$$

3　数值实验

为验证格式的精度，引入以下记号

$$r_1=\log_2\frac{E(h,\tau)}{E(h/2,\tau)}, r_2=\log_2\frac{E(h,\tau)}{E(h,\tau/2)},$$

其中 $E(h,\tau)$ 为在时空步长分别为 h、τ 时的 $\|e^N\|$.

例 1　考虑如下初边值问题

$$\begin{cases}\mathrm{i}u_t+u_{xx}-(|u|^2+|u|^4)u=f(x,t)u, 15<x<15, 0<t\leqslant 1,\\ u(15,t)=u(15,t)=0, 0<t\leqslant 1,\\ u_0(x)=u(x,0)=\exp(-x^2+\mathrm{i}x), -15\leqslant x\leqslant 15,\end{cases}$$

其中 $f(x,t)=4(x-2t)^2-\exp[-2(x-2t)^2]-\exp[-4(x-2t)^2]$.

该问题的精确解为 $u(x,t)=\exp(-(x-2t)^2+\mathrm{i}(x-3t))$.

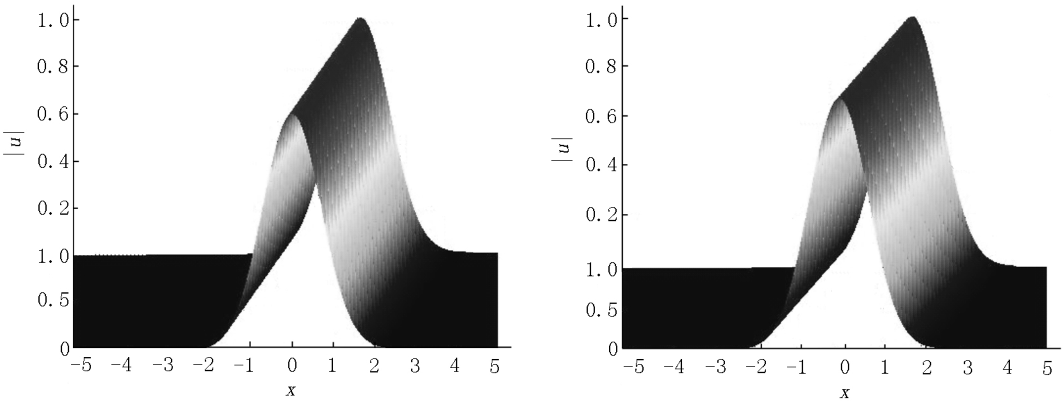
在验证空间方向(时间方向)的收敛阶时，取 $\tau=0.000\,1$ ($h=0.001$)，这样可以忽略时间(空间)方向的误差影响. 表 1 和表 2 分别给出了空间和时间方向的收敛阶. 在表 3 中，将本文的线性格式与文献[20]中的非线性格式做了计算效率上的比较. 图 1 展示了精确解和数值解在不同时间层下的波形变化.

表 1　空间方向收敛阶

τ	h	$E(h,\tau)$	r_1
0.000 1	0.40	8.4000e-03	4.134 0
0.000 1	0.20	4.7842e-04	4.018 0
0.000 1	0.10	2.9530e-05	4.028 9
0.000 1	0.05	1.8090e-06	—

表 2　时间方向收敛阶

h	τ	$E(h,\tau)$	r_2
0.001	0.002 00	1.7017e-04	1.997 9
0.001	0.001 00	4.2603e-05	1.999 0
0.001	0.000 50	1.0658e-05	1.999 4
0.001	0.000 25	2.6656e-06	—



(a) 精确解　　(b) 数值解

图 1　算例在 $\tau=0.001$, $h=0.1$ 时的解

表 3 线性格式(LCFD)与非线性格式(NCFD)在计算效率上的比较

h	τ	Scheme	CPU time/s
0.000 3	0.100 00	NCFD	13.95
0.000 3	0.100 00	LCFD	10.33
0.000 3	0.050 00	NCFD	29.21
0.000 3	0.050 00	LCFD	20.18
0.400 0	0.000 01	NCFD	204.83
0.400 0	0.000 01	LCFD	78.78
0.200 0	0.000 01	NCFD	481.12
0.200 0	0.000 01	LCFD	157.09

由以上数值实验的结果可看出,差分格式(2)~(6)在时空方向分别具有 2 阶和 4 阶精度,这完全符合定理 1 的结论. 由表 2 和表 3 可见,本文格式有较好的稳定性,而且与已有格式相比,在精度相当的前提下计算效率有大幅提高.

4 参考文献

[1] 张鲁明,常谦顺. 带 5 次项的非线性 Schrödinger 方程的守恒数值格式[J]. 应用数学,1999,12(1):65-71.

[2] 张鲁明,常谦顺. 带 5 次项的非线性 Schrödinger 方程差分解法[J]. 应用数学学报,2000,23(3):351-358.

[3] Bao Weizhu,Jaksch D,Markowich P A. Numerical solution of the Gross-Pitaevskii equation for Bose-Einstein condensation[J]. J Comput Phys,2003,187(1):318-342.

[4] Bao Weizhu, Li Hailiang, Shen Jie. A generalized-Laguerre-Fourier-Hermite pseudospectral method for computing the dynamics of rotating Bose-Einstein condensates[J]. SIAM Journal on Scientific Computing,2009,31(5):3685-3711.

[5] Bao Weizhu, Shen Jie. A fourth-order time-splitting Laguerre-Hermite pseudo-spectral method for Bose-Einstein condensates[J]. SIAM Journal on Scientific Computing,2005,26(6):2010-2028.

[6] Argyris J,Haase M. An engineer´s guide to soliton phenomena: application of the finite element method[J]. Computer Methods in Applied Mechanics and Engineering,1987,61(1):71-122.

[7] Akrivis G D,Dougalis V A,Karakashian O A. On fully discrete Galerkin methods of second-order temporal accuracy for the nonlinear Schrödinger equation[J]. Numerische Mathematik,1991,59(1):31-53.

[8] Karakashian O,Makridakis C. A space-time finite element method for the nonlinear Schrödinger equation: the discontinuous Galerkin method[J]. Mathematics of Computation,1998,67(222):479-499.

[9] Xu Yan,Shu Chiwang. Local discontinuous Galerkin methods for nonlinear Schrödinger equations[J]. Journal of Computational Physics,2005,205(1):72-97.

[10] Dehghan M,Mirzaei D. Numerical solution to the unsteady two-dimensional Schrödinger equation using meshless local boundary integral equation method[J]. International Journal for Numerical Methods in Engineering,2008,76(4):501-520.

[11] Dehghan M,Mirzaei D. The meshless local Petrov-Galerkin (MLPG) method for the generalized two-dimensional nonlinear Schrödinger equation[J]. Engineering Analysis with Boundary Elements,2008,32(9):747-756.

[12] Taha T R,Ablowitz M J. Analytical and numerical aspects of certain nonlinear evolution equations: numerical nonlinear Schrödinger equation[J]. Journal of Computational Physics,1984,55(2):203-2230.

[13] Cloot A,Herbst B M,Weideman J A C. A numerical study of the nonlinear Schrödinger equation involving quintic terms[J]. Journal of Computational Physics,1990,86(1):127-146.

[14] Dehghan M,Taleei A. A compact split-step finite difference method for solving the nonlinear Schrödinger equations with constant and variable coefficients[J]. Computer Physics Communications,2010,181(1):43-51.

[15] 王廷春,郭柏灵. 一维非线性 Schrödinger 方程的两个无条件收敛的守恒紧致差分格式[J]. 中国科学: 数学,2011,41(3):207-233.

[16] Sanz-Sema J M. Methods for the numerical solution of the nonlinear Schrödinger equation[J]. Math Comp,1984,43(167):21-27.

[17] Chang Qianshun, Jia Erhui, Sun Weiwei. Difference schemes for solving the generalized nonlinear Schrödinger equation[J]. J Comput Phys,1999,148(2):397-415.

[18] Delfour M,Fortin M,Payre G. Finite-difference solution of a non-llinear Schrödinger equation[J]. J Comput Phys,1981,44(2):277-288.

[19] 王询,曹圣山. 带 5 次项的非线性 Schrödinger 方程新差分格式[J]. 中国海洋大学学报: 自然科学版,2009,39(增刊):487-491.

[20] 初日辉. 带 5 次项的非线性 Schrödinger 方程的一个紧致差分格式[J]. 江苏师范大学学报: 自然科学版,2014,32(2):53-57.

[21] Samarskii A A,Andreev V B. Difference methods for elliptic equations[M]. Moscow: Nauka Izdat,1976.

[22] Wang Tingchun,Zhao Xiaofei. Optimal l∞ error estimates of finite difference methods for the coupled Gross-Pitaevskii equations in high dimensions[J]. Science China Mathematics,2014,57(10):2189-2214.

(下转第 51 页)

The Mechanism and Empirical Study on the Effect of Industrial Structure Optimization on the Development of Logistics Industry

LIU Lu[1] ,LI Shengsheng[2] ,ZHOU Yunlei[3]

(1. College of Modern Economics and Management,Jiangxi University of Finance and Economics,Gongqingcheng Jiangxi 332020,China;
2. School of Information Management,Jiangxi University of Finance and Economics,Nanchang Jiangxi 330013,China;
3. School of Economics,Anhui University,Hefei Anhui 230601,China)

Abstract: The mechanism of industrial structure optimization on the development of logistics industry from two aspects of conduction path and mathematical model is analyzed. In order to verify the assumptions made by the mathematical model,two indicators have been constructed for optimizing the industrial structure,which are the advanced industrial structure and the rationalization of the industrial structure. And using panel data from 31 provinces and cities from 1986 to 2015,the relationship between industrial structure optimization and logistics industry development level is empirically studied. Research shows that the advanced industrial structure significantly promotes the development of the logistics industry,the rationalization of industrial structure also plays a promoting role to a certain extent. On this basis,a dynamic panel model is constructed and the results are in agreement with the above conclusions,but there are significant differences in various regions.

Key words: optimization of industrial structure; logistics development; mechanism; dynamic panel model

(责任编辑:曾剑锋)

(上接第 38 页)

The Linearized Four-Level Compact Finite Difference Scheme for the Quintic Nonlinear Schrödinger Equation

ZHAI Buxiang[1] ,NIE Tao[1*] ,XUE Xiang[2]

(1. Basic Department,Nanjing Polytechnic Institute,Nanjing Jiangsu 210048,China;
2. School of Mathematics and Statistics,Nanjing University of Information Science and Technology,Nanjing Jiangsu 210044,China)

Abstract: In this paper,a linearized four-level compact finite difference scheme for the nonlinear Schrödinger equation involving quintic term is proposed. By introducing a "lifting" technique,the optimal error estimate is established by using standard energy method and mathematical induction. It is proved that the numerical solution has fourth-order and second-order accuracy in space and in time,respectively. Numerical experiments are given to verify the theoretical results and compared with the existing results. The results show that the proposed scheme has higher computational efficiency under the condition of maintaining the accuracy.

Key words: quintic nonlinear Schrödinger equation; compact finite difference scheme; optimal error estimate; linearized four-level scheme; computational efficiency

(责任编辑:曾剑锋)