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离散时滞动力学网络的拓扑辨识

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摘要:考虑离散时滞动力学网络的拓扑辨识问题. 通过选取合适的参数自适应更新法则,设计有效的响应网络估计器来辨识网络中未知的或不确定的耦合矩阵,并利用 Frobenius 矩阵范数和 Lasalle 不变原理证明了该方法的可行性. 值得注意的是,网络的耦合矩阵可以是有向加权的. 2 个数值例子验证了该方法的有效性.

关键词:拓扑辨识;离散时滞;动力学网络

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0 引言

网络科学在过去 20 年里吸引了来自各个领域研究者们的关注[1-6],通过建立相应的动力学网络模型来描述并分析物理、生物、社会关系等方面的现象. 研究者们在研究网络的结构属性、动力学行为、同步控制时,一般假设拓扑结构已知. 尤其在牵制控制和节点的重要性排序问题上,网络的拓扑结构通常起着重要作用. 牵制控制是指有选择地对网络中的小部分节点加以控制进而使得整个网络达到期望行为. 选取节点的标准一般包括节点度的大小、节点间的连接关系、网络的社团结构等[7-12],这都要求网络的拓扑结构是已知的. 节点的重要性排序方法主要有基于近邻排序、基于路径排序、基于特征向量排序等[13-17],以上方法同样是在网络拓扑结构已知的情况下提出的. 由此可见,辨识动力学网络的拓扑结构是一个重要课题[18-27].

Yu Dongchuan 等[18]首先提出了一种基于动力学演变的方法来估计连续时间网络的拓扑结构. 该方法在有外界干扰和模型误差时依然适用. 随后,研究者们在连续时间网络的结构辨识问题上取得了一系列成果[19-25]. 其中 Wu Xiaoqun 等[19-21]研究了连续时滞网络的结构辨识问题. 另一方面,Guo Shujuan 等[26-28]研究了离散时间网络的结构辨识问题. 事实上,在对连续时间网络进行图像处理、模式识别和数值模拟时,需要先将其离散化[29]. 因此,研究离散时间网络具有直接的理论意义和重要的实际价值[30-35].

结合以上讨论,同时考虑现实网络常存在时滞现象[36-39],如直播比赛时的信号时滞、人体感染细菌后的发病时滞等,本文研究离散时滞动力学网络的拓扑辨识问题,提出了一种有效的方法来辨识拓扑结构,并通过 Frobenius 矩阵范数和 Lasalle 不变原理证明了该方法的可行性.

1 预备知识

1.1 矩阵的迹

记 $M_{m\times n}(\mathbf{R})$ 为实数域 $\mathbf{R}$ 上 $m\times n$ 维矩阵的集合,有以下结论:

(i) $\forall \boldsymbol{A}=(a_{ij})\in M_{n\times n}(\mathbf{R})$,有
$$\mathrm{tr}(\boldsymbol{A})=\mathrm{tr}(\boldsymbol{A}^{\mathrm{T}})=\sum_{i=1}^{n}a_{ii};$$

(ii) $\forall \boldsymbol{A},\boldsymbol{B}\in M_{n\times n}(\mathbf{R})$,$\forall\alpha,\beta\in\mathbf{R}$,有
$$\mathrm{tr}(\alpha\boldsymbol{A}+\beta\boldsymbol{B})=\alpha\mathrm{tr}(\boldsymbol{A})+\beta\mathrm{tr}(\boldsymbol{B});$$

(iii) $\forall \boldsymbol{A}\in M_{m\times n}(\mathbf{R})$,$\forall \boldsymbol{B}\in{}_{n\times m}(\mathbf{R})$,有
$$\mathrm{tr}(\boldsymbol{AB})=\mathrm{tr}(\boldsymbol{BA});$$

(iv) $\forall \boldsymbol{A}=(a_{ij})\in M_{m\times n}(\mathbf{R})$,有
$$\mathrm{tr}(\boldsymbol{AA}^{\mathrm{T}})=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}^{2}.$$

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1.2 Frobenius 矩阵范数

选取 $A,B\in M_{m\times n}(\mathbf{R})$,若 A,B 分别表示列向量,$\mathrm{tr}(AB^{\mathrm{T}})$ 是空间 $M_{m\times n}(\mathbf{R})$ 上的内积,则由这种内积推导的矩阵范数被称为 Frobenius 矩阵范数(简称 F 范数),记为 $\|\cdot\|_F$,有以下结论[40]:

(i) $\forall \boldsymbol{A}=(a_{ij})\in M_{m\times n}(\mathbf{R})$,有

$$\|\boldsymbol{A}\|_F=(\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}))^{1/2};$$

(ii) $\forall \boldsymbol{A}=(a_{ij})\in M_{m\times n}(\mathbf{R}),\forall \boldsymbol{B}=(b_{jk})\in M_{n\times p}(\mathbf{R})$,有 $\|\boldsymbol{AB}\|_F\leqslant\|\boldsymbol{A}\|_F\|\boldsymbol{B}\|_F$.

1.3 Lasalle 不变原理

考虑差分方程

$$\boldsymbol{x}(t+1)=S(\boldsymbol{x}(t)),t=0,1,2,\cdots,\quad(1)$$

其中 $S:\mathbf{R}^n\to\mathbf{R}^n$.

若(i) $G\subset\mathbf{R}^n$;(ii) V 是连续的且 $\forall\boldsymbol{x}(t)\in G$ 均有 $V(\boldsymbol{x}(t))\geqslant 0$ 成立;(iii) $V(\boldsymbol{x}(t))=0$ 当且仅当 $\boldsymbol{x}(t)=\boldsymbol{0}$,则称 V 是方程(1)的 Lyapunov 函数.

记 $E=\{\boldsymbol{x}:\Delta V(\boldsymbol{x}(t))=0,\boldsymbol{x}\in G_1\}$,其中 G_1 是 G 的闭包. 令 $M\subseteq E$,且 $S(M)=M$,若 $M'\subseteq E$ 且 $S(M')=M'$,有 $M'\subseteq M$,则称 M 为 E 的最大不变集. 记 $V^{-1}(c)=\{\boldsymbol{x}:V(\boldsymbol{x}(t))=c,\boldsymbol{x}\in\mathbf{R}^n\}$.

Lasalle 不变原理[27]:若(i) V 是 G 上方程(1)的 Lyapunov 函数且 $\Delta V(\boldsymbol{x}(t))\leqslant 0$ 成立;(ii) $\boldsymbol{x}(t)$ 是 G 上 $\forall t\geqslant 0$ 的方程(1)的有界解,则存在一个数 c 使得当 $t\to\infty$ 时,有 $\boldsymbol{x}(t)\to M\cap V^{-1}(c)$ 成立.

2 模型描述

考虑离散时滞动力学网络

$$\boldsymbol{x}_i(t+1)=f_i(\boldsymbol{x}_i(t))+\sum_{j=1}^{N}a_{ij}\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)),$$
$$i=1,2,\cdots,N,\quad(2)$$

其中 $\boldsymbol{x}_i=(x_{i1},x_{i2},\cdots x_{in})^{\mathrm{T}}\in\mathbf{R}^n$ 是节点 i 的状态向量,$f_i(\boldsymbol{x}_i):\mathbf{R}^n\to\mathbf{R}^n$ 是节点 i 的局部动力学方程,$\boldsymbol{h}_j(\boldsymbol{x}_j)=(h_{j1}(\boldsymbol{x}_j),h_{j2}(\boldsymbol{x}_j),\cdots,h_{jn}(\boldsymbol{x}_j))^{\mathrm{T}}$ 是节点 j 的输出方程,$\tau\in\mathbf{Z}^+$ 表示时滞. $\boldsymbol{A}=(a_{ij})\in\mathbf{R}^{N\times N}$ 为耦合矩阵,它表示网络的拓扑结构和耦合强度,定义为:当节点 $j(j\neq i)$ 对节点 i 有影响时 $a_{ij}\neq 0$,否则 $a_{ij}=0$,且满足 $a_{ii}=-\sum\limits_{j=1,j\neq i}^{N}a_{ij}$.

在网络(2)中,假设函数 f_i 和 $\boldsymbol{h}_j$ 是已知的,$\boldsymbol{x}_i(t)$ 可观测的,而网络的拓扑结构是未知的.

假设1 函数 $f_{ik}(i=1,2,\cdots,N,k=1,2,\cdots,n)$ 有界.

假设2 函数 h_{jk} 有界,即存在正数 L_{jk},使得 $\forall\boldsymbol{x}_j(t)\in\mathbf{R}^n$ 有

$$|h_{jk}(\boldsymbol{x}_j)|\leqslant L_{jk}(k=1,2,\cdots,n,j=1,2,\cdots,N).$$

假设3 $\{\boldsymbol{h}_j(\boldsymbol{x}_j)\}_{j=1}^{N}$ 关于 $\boldsymbol{x}_j(j=1,2,\cdots,N)$ 是线性独立的.

3 拓扑辨识

为了辨识拓扑结构未知的网络(2),建立响应网络

$$\boldsymbol{y}_i(t+1)=f_i(\boldsymbol{x}_i(t))+\sum_{j=1}^{N}b_{ij}(t)\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)),i=1,2,\cdots,N,\quad(3)$$

其中 $\boldsymbol{y}_i=(y_{i1},y_{i2},\cdots,y_{in})^{\mathrm{T}}\in\mathbf{R}^n$,$\boldsymbol{B}(t)=(b_{ij}(t))\in\mathbf{R}^{N\times N}$ 表示随时间变化的参数矩阵,用于辨识未知矩阵 $\boldsymbol{A}$.

记 $\boldsymbol{e}_i(t)=\boldsymbol{y}_i(t)-\boldsymbol{x}_i(t)$ 为系统误差,则

$$\boldsymbol{e}_i(t)=\sum_{j=1}^{N}(b_{ij}(t)-a_{ij})\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)).$$

定理1 若假设1~假设3成立. 参数矩阵 $\boldsymbol{B}(t)=(b_{ij}(t))$ 可以通过以下自适应更新法则来辨识网络(2)的未知矩阵 $\boldsymbol{A}=(a_{ij})$:

$$b_{ij}(t+1)=b_{ij}(t)-k\boldsymbol{e}_i(t+1)^{\mathrm{T}}\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)),$$
$$i,j=1,2,\cdots,N,\quad(4)$$

其中 $k>0$ 为常数.

证 根据假设1和假设2,响应网络(3)的解有界.

首先将方程(2)~(4)写成矩阵形式,得到

$$\boldsymbol{X}(t+1)=\boldsymbol{F}(\boldsymbol{X}(t))+\boldsymbol{AH}(\boldsymbol{X}(t-\tau)),\quad(5)$$
$$\boldsymbol{Y}(t+1)=\boldsymbol{F}(\boldsymbol{X}(t))+\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau)),\quad(6)$$
$$\boldsymbol{B}(t+1)=\boldsymbol{B}(t)-k\boldsymbol{E}(t+1)\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}},\quad(7)$$

其中

$$\boldsymbol{X}(t)=(x_1(t),x_2(t),\cdots,x_N(t))^{\mathrm{T}}\in\mathbf{R}^{N\times n},$$
$$\boldsymbol{Y}(t)=(y_1(t),y_2(t),\cdots,y_N(t))^{\mathrm{T}}\in\mathbf{R}^{N\times n},$$
$$\boldsymbol{E}(t)=(e_1(t),e_2(t),\cdots,e_N(t))^{\mathrm{T}}\in\mathbf{R}^{N\times n},$$
$$\boldsymbol{F}(\boldsymbol{X})=(f_1(x_1),f_2(x_2),\cdots,f_N(x_N))^{\mathrm{T}}\in\mathbf{R}^{N\times n},$$
$$\boldsymbol{H}(\boldsymbol{X})=(h_1(x_1),h_2(x_2),\cdots,h_N(x_N))^{\mathrm{T}}\in\mathbf{R}^{N\times n},$$

(6)式减去(5)式得到

$$\boldsymbol{E}(t+1)=(\boldsymbol{B}(t)-\boldsymbol{A})\boldsymbol{H}(\boldsymbol{X}(t-\tau)),\quad(8)$$

把(8)式代入(7)式,两边分别减去 $\boldsymbol{A}$ 得到

$$\Delta\boldsymbol{B}(t+1)=\Delta\boldsymbol{B}(t)[\boldsymbol{I}-k\boldsymbol{H}(\boldsymbol{X}(t-\tau))\cdot\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}}],$$

其中 $\Delta\boldsymbol{B}(t)=\boldsymbol{B}(t)-\boldsymbol{A}$,$\boldsymbol{I}$ 是单位矩阵.

构造如下 Lyapunov 函数:

$$V(t)=\|\Delta\boldsymbol{B}(t)\|_F^2,$$

根据 F 范数的相关结论,有

$$\begin{aligned}\Delta V &= V(t+1)-V(t) = \|\Delta\boldsymbol{B}(t)[\boldsymbol{I}-k\boldsymbol{H}(\boldsymbol{X}(t-\tau))\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}}]\|_F^2-\|\Delta\boldsymbol{B}(t)\|_F^2 = -2k\cdot\\ &\mathrm{tr}[(\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau)))(\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau)))^{\mathrm{T}}]+\\ &k^2\mathrm{tr}[(\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}})\cdot(\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}})^{\mathrm{T}}] =\\ &-2k\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2+k^2\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\boldsymbol{H}(\boldsymbol{X}(t-\tau))^{\mathrm{T}}\|_F^2 \leqslant -2k\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2+\\ &k^2\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2\|\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2 = -k(2-k\|\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2)\cdot\\ &\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2.\end{aligned}$$

根据假设 2,通过选择 $0<k<2(\sum_{j=1}^{N}\sum_{k=1}^{n}L_{jk}^2)^{-1}$,可以使得 $-k(2-k\|\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2)<0$. 由于 $\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2\geqslant 0$,所以有 $\Delta V\leqslant 0$.

令 $\Delta V=0$,得到 $\|\Delta\boldsymbol{B}(t)\boldsymbol{H}(\boldsymbol{X}(t-\tau))\|_F^2=0$,即

$$\sum_{j=1}^{N}\Delta b_{ij}(t)h_{jk}(x(t-\tau)) = 0, i=1,2,\cdots,N,$$
$$k=1,2,\cdots,n,$$

或者

$$\sum_{j=1}^{N}\Delta b_{ij}(t)\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)) = 0, i=1,2,\cdots,N. \quad (9)$$

据假设 3,$\{\boldsymbol{h}_j(\boldsymbol{x}_j)\}_{j=1}^{n}$ 是线性独立的,那么从方程(9) 中可以得出 $\Delta b_{ij}(t)=0$,故 $\forall i,j=1,2,\cdots,N$,有 $\boldsymbol{e}_i(t)=\boldsymbol{0}$. 根据 Lasalle 不变原理,$\Delta b_{ij}(t)=0$ 是 $\Delta V(t)=0$ 的最大不变集,即 $b_{ij}(t)=a_{ij}(i,j=1,2,\cdots,N)$ 是(4) 式的全局渐近吸引子.

4　例子

本节通过 2 个数值模拟的例子来证明定理 1 的正确性和有效性.

例 1　考虑由 6 个节点组成的网络

$$\boldsymbol{x}_i(t+1) = \boldsymbol{f}_i(\boldsymbol{x}_i(t)) + \sum_{j=1}^{6}a_{ij}\boldsymbol{h}_j(\boldsymbol{x}_j(t-\tau)),$$
$$i=1,2,\cdots,6, \quad (10)$$

每个节点具有如下 3 维 Lorenz 映射动力学[41]:

$$\begin{cases}x_{i1}(t+1) = (1-\alpha T)x_{i1}(t)+\alpha Tx_{i2}(t),\\ x_{i1}(t+1) = \beta Tx_{i1}(t)-Tx_{i1}(t)x_{i3}(t)+(1-T)x_{i2}(t),\\ x_{i3}(t+1) = Tx_{i1}(t)x_{i2}(t)+(1-\gamma T)x_{i3}(t),\end{cases}$$

其中 $\alpha=10,\beta=28,\gamma=10/3,T=0.01$,$|x_{i1}|\leqslant 25$,$|x_{i2}|\leqslant 30$,$|x_{i3}|\leqslant 55$.

在数值模拟中,选取 $\tau=2,\varepsilon=0.005,\boldsymbol{h}_j(\boldsymbol{x}_j(t))=(\varepsilon x_{j1}(t),\varepsilon x_{j2}(t),\varepsilon x_{j3}(t))^{\mathrm{T}}$,则有 $|h_{j1}|\leqslant L_{j1}=25\varepsilon$,$|h_{j2}|\leqslant L_{j2}=30\varepsilon$,$|h_{j3}|\leqslant L_{j3}=55\varepsilon$,则 $2(\sum_{j=1}^{6}\sum_{k=1}^{3}L_{jk}^2)^{-1}=2.9304$,即当 $0<k<2.9304$ 时(取 $k=2.5$),可用 $b_{ij}(t)$ 辨识 a_{ij}. 选取网络(10) 的拓扑结构如下:

$$\boldsymbol{A}=(a_{ij})=\begin{bmatrix}-3&1&0&1&0&1\\1&-3&1&0&0&1\\0&1&-3&1&1&0\\1&0&1&-3&1&0\\0&0&1&1&-3&1\\1&1&0&0&1&-3\end{bmatrix},$$

图 1 显示可以有效辨识出未知的拓扑结构.

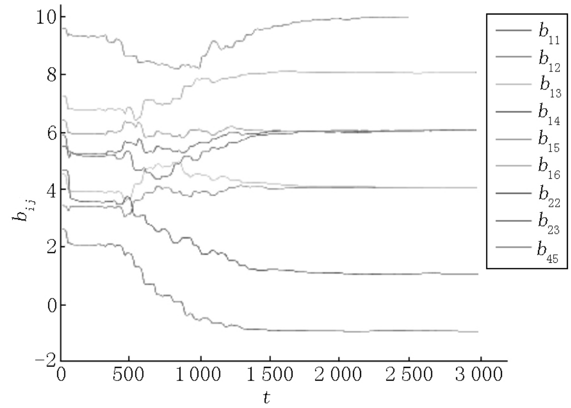

图 1　拓扑结构辨识

例 2　在例 1 的网络中,将矩阵 $\boldsymbol{A}$ 改有向加权矩阵,其他条件不变,用 $b_{ij}(t)$ 辨识 a_{ij} 的效果如图 2 所示.

$$\boldsymbol{A}=(a_{ij})=\begin{bmatrix}-3&1&2&-1&0&1\\0&-2&3&0&-2&1\\0&2&-2&1&-1&0\\1&0&1&-4&1&3\\2&3&1&-2&-5&1\\1&2&1&-4&1&-1\end{bmatrix}.$$

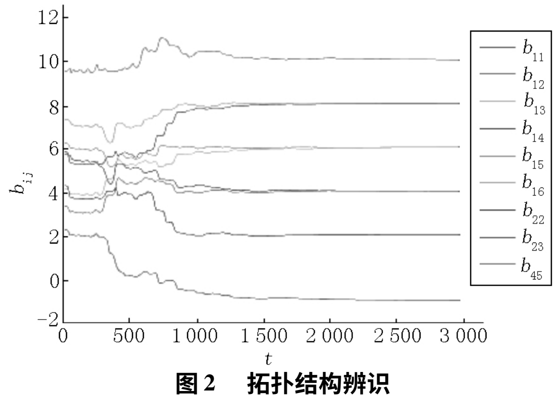

图 2　拓扑结构辨识

5　结论

本文提出了一种有效方法来辨识离散时滞动力

学网络的拓扑结构,并利用 Frobenius 矩阵范数和 Lasalle 不变原理证明了该方法的可行性. 该方法通过建立 $B(t)$ 的自适应更新法则来辨识 A. 耦合矩阵可以是有向加权矩阵,2 个例子验证了这一方法的正确性和有效性.

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The Topology Identification of Discrete-Time Dynamical Network with Delay

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Abstract: Topology identification of discrete-time dynamical network with delay is considered. The effective response network estimator through choosing proper parameter adaptive updating laws is designed to identify unknown or uncertain coupling matrix. The method is proved to be effective according to Frobenius matrix norm and Lasalle′s invariance principle. It′s worth noting that the coupling matrix of network can be directed and weighted. The method is verified by two examples.

Key words: topology identification; discrete-time; dynamical networks

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