文章编号: 1000-5862(2019)02-0122-06

Some Generalizations and Improvements of Montel´s Criterion

XU Yan
(Institute of Mathematics, School of Mathematics, Nanjing Normal University, Nanjing Jiangsu 210023, China)

Abstract: Montel´s criterion is the most celebrated theorem in the theory of normal families of meromorphic functions, and plays an important role in value distribution theory, complex dynamics, extremal problems, etc. Up to now, there have been a variety of extensions and improvements of Montel´s criterion. In this paper, some generalizations and improvements of Montel´s criterion are summarized. Also, two questions related to Montel´s criterion are given.

Key words: Montel´s criterion; normal family; exceptional value; shared value
中图分类号: O 174.52 **文献标志码:** A **DOI:** 10.16357/j.cnki.issn1000-5862.2019.02.02

0 Montel´s Criterion

Let D be a domain in the complex plane $\mathbf{C}$, a family F of functions meromorphic on D is normal, in the sense of Montel, if every sequence $\{f_n\} \in F$ contains a subsequence $\{f_{n_j}\}$ which converges spherically uniformly on compact of D[1-3].

The most celebrated theorem in the theory of normal families is the following criterion of Montel[4], which is the local counterpart of Picard theorem.

Montel´s Criterion Let F be a family of functions meromorphic on a domain D, and let a_1, a_2 and a_3 be three distinct complex numbers in $\hat{\mathbf{C}}\ (= \mathbf{C} \cup \{\infty\})$. If, for each $f \in F$, f omits a_1, a_2, a_3 in D, then F is normal in D.

Obviously, two exceptional values are sufficient for holomorphic families, since each holomorphic function omits ∞ naturally. Montel´s criterion is called the Fundamental Normality Test in the book[2]. From it flow effortlessly the classical theorems of Picard, Schottky, and Julia, as well as various covering theorems. It plays an important role in complex dynamics, extremal problems, etc.

1 It´s Generalizations and Improvements

Up to now, there have been a variety of extensions and improvements of Montel´s criterion. Generally speaking, there are two directions to generalize or improve Montel´s criterion. One is replacing "fixed" exceptional values by "moving" exceptional values (exceptional functions and ´wandering´ exceptional functions), the other is allowing $f \in F$ to have exceptional values under certain conditions. Of course, one can combine these two directions to generalize or improve Montel´s criterion.

Here we say two functions $a(z)$ and $b(z)$ avoiding each other in D if for each $z \in D$, $a(z) \neq b(z)$, and functions $a(z)$ and $b(z)$ are called distinct if there exists $z \in D$ such that $a(z) \neq b(z)$.

The next theorem generalized Montel´s criterion, by extending three fixed values a_1, a_2, a_3 to meromor-

收稿日期: 2018-11-28
基金项目: 国家自然科学基金(11471163)资助项目.
作者简介: 徐 焱(1964-),男,江苏镇江人,教授,博士生导师,主要从事亚纯函数值分布正规族与唯一性等方面的研究.
E-mail: xuyan@njnu.edu.cn

phic functions $a_1(z), a_2(z), a_3(z)$ avoiding each other in D.

Theorem 1 [2] Let F be a family of functions meromorphic on a domain D, and let $a_1(z), a_2(z)$ and $a_3(z)$ be three meromorphic functions avoiding each other in D. If, for each $f \in F$, $f(z) \neq a_i(z)$ $(i = 1, 2, 3)$ in D, then F is normal in D.

Remark 1 This observation seems first to have been made by Fatou.

To see this, we only consider, instead of F, the family

$$\mathscr{G} = \left\{ g(z) = \frac{f(z) - a_1(z)}{f(z) - a_2(z)} \frac{a_3(z) - a_2(z)}{a_3(z) - a_1(z)} \right\}.$$

All $g \in \mathscr{G}$ omit the values $0, 1$ and ∞ in D. Then by Montel's criterion, $\mathscr{G}$ is normal, so that F is also normal.

Remark 2 Bargmann-Bonk-Hinkkanen-Martin [5] proved that Montel's criterion is still valid if a_1, a_2, a_3 are replaced by three arbitrary continuous functions avoiding each other in D.

Chang-Fang-Zalcman [6] improved Theorem 1 and obtained

Theorem 2 Let F be a family of functions meromorphic on a domain D, and let $a_1(z), a_2(z)$ and $a_3(z)$ be distinct meromorphic functions in D. If, for each $f \in F$, $f(z) \neq a_i(z)$ $(i = 1, 2, 3)$ in D, then F is normal in D.

The following significant improvement is proved by Montel [2] himself, which allow f to have values a_1, a_2, a_3 but restricting their multiplicities.

Theorem 3 Let F be a family of functions meromorphic on a domain D, a_1, a_2, a_3 be three distinct complex numbers in $\hat{\mathbf{C}}$, and l_1, l_2, l_3 be positive integers or ∞. If, for each $f \in F$, all zeros of $f - a_i$ have multiplicity at least l_i for $i = 1, 2, 3$ with

$$1/l_1 + 1/l_2 + 1/l_3 < 1,$$

then F is normal in D.

It is natural to consider: What can we say if three distinct complex numbers a_1, a_2 and a_3 in Theorem 3 are replaced by three distinct meromorphic functions $a_1(z), a_2(z)$ and $a_3(z)$ in D? In [7 (cf. [8])], Xu Yan proved the following result.

Theorem 4 Let F be a family of functions meromorphic on a domain D, let $a_1(z), a_2(z)$ and $a_3(z)$ be three distinct meromorphic functions in D, one of which may be ∞ identically, and let l_1, l_2 and l_3 be positive integers or ∞ with

$$1/l_1 + 1/l_2 + 1/l_3 < 1.$$

Suppose that, for each $f \in F$ and $z \in D$,

(i) all zeros of $f - a_i$ have multiplicity at least l_i for $i = 1, 2, 3$;

(ii) $f(z_0) \neq a_i(z_0)$ if there exist $i, j \in \{1, 2, 3\}$ $(i \neq j)$ and $z_0 \in D$ such that $a_i(z_0) = a_j(z_0)$,

then F is normal in D.

Remark 3 Condition (ii) cannot be omitted in Theorem 4, as is shown by the following examples.

Example 1 Let $D = \{z: |z| < 1\}$, $a_1(z) = 0$, $a_2(z) = \infty$, and $a_3(z) = z^k$, where $k \geqslant 3$ is a positive integer, and

$$F = \{f_n(z) = nz^3, n = 2, 3, \cdots, z \in D\}.$$

Since $f_n(z) - a_3(z) = z^3(n - z^{k-3})$, $l_1 = 3$, $l_2 = \infty$, and $l_3 = 3$, then condition (i) in Theorem 4 is satisfied. But F is not normal in D. Note that $f_n(0) = a_1(0) = a_3(0)$.

Example 2 Let $D = \{z: |z| < 1\}$, $a_i(z) = i/z^k$ $(i = 1, 2, 3)$, where $k \geqslant 1$ is a positive integer, and

$$F = \{f_n(z) = 1/nz, n = 2, 3, \cdots, z \in D\}.$$

Since

$$f_n(z) - a_i(z) = (z^{k-1} - in)/(nz^k) \neq 0$$

in D, $l_1 = l_2 = l_3 = \infty$, and thus condition (i) in Theorem 4 is satisfied. But F is not normal in D. Note that $f_n(0) = a_1(0) = a_2(0) = a_3(0)$.

Remark 4 Letting $l_1 = l_2 = l_3 = \infty$, it is easy to see that Theorem 2 is a direct consequence of Theorem 4.

In an analogous fasion as in Theorem 3, A. Bloch [9] and G. Valiron [10] proved the next more general result.

Theorem 5 Let F be a family of functions meromorphic on a domain D, a_i $(i = 1, 2, \cdots, q)$ be distinct complex numbers in $\hat{\mathbf{C}}$, and l_i $(i = 1, 2, \cdots, q)$ be positive integers or ∞. For each $f \in F$, all zeros of $f - a_i$ have multiplicity at least l_i for $i = 1, 2, \cdots, q$. If $\sum_{i=1}^{q} (1 -$

$1/l_i) > 2$, then F is normal in D.

One can ask

Question 1 Does Theorem 5 still hold if distinct complex numbers $a_i(i=1,2,\cdots,q)$ are replaced by distinct meromorphic functions $a_i(z)(i=1,2,\cdots,q)$ in D?

On the other hand, instead of fixed exceptional values, C. Carathéodoary[11] considered exceptional values depending on the respective functions in the family under consideration.

Theorem 6 Let F be a family of functions meromorphic on a domain D. Suppose there exists an $\varepsilon > 0$ such that each $f \in F$ omits three values $a_f, b_f, c_f \in \hat{\mathbf{C}}$ satisfying

$$\min\{\chi(a_f,b_f),\chi(a_f,c_f),\chi(b_f,c_f)\} \geqslant \varepsilon,$$

where χ denotes the spherical metric on $\hat{\mathbf{C}}$. Then F is normal in D.

Grahl-Nevo[12] extended Carathéodoary's result futher, by considering meromorphic exceptional functions which also depend on the respective functions in the family under consideration, i. e., kind of 'wandering' exceptional functions.

Theorem 7 Let F be a family of functions meromorphic on a domain D, $\varepsilon > 0$. Assume that for each $f \in F$ there exist meromorphic functions $a_f(z)$, $b_f(z)$, $c_f(z)$ such that f omits $a_f(z)$, $b_f(z)$, $c_f(z)$ in D, and

$$\min\{\chi(a_f(z),b_f(z)),\chi(a_f(z),c_f(z)),\chi(b_f(z),c_f(z))\} \geqslant \varepsilon$$

for all $z \in D$. Then F is normal in D.

Remark 5 Here the spherical metric is crucial. Indeed, it is not true for the Euclidean metric since could choose $a_f = f+1$, $b_f = f+2$, $c_f = f+3$ for any $f \in F$. In [12], they gave an example to show that Theorem 7 no longer holds for 'wandering' continuous exceptional functions.

Let f,g be two meromorphic functions on a domain D, $a \in \hat{\mathbf{C}}$, and S be a non-empty subset of $\hat{\mathbf{C}}$. We say that f and g share the value a in D if $\overline{E}_f(a,D) = \overline{E}_g(a,D)$, where $\overline{E}_f(a,D) = \{z \in D: f(z) - a = 0\}$, where each multiple zeros is only counted one time (ignoring multiplicity). We say that f and g share the set S in D if $\overline{E}_f(S,D) = \overline{E}_g(S,D)$, where $\overline{E}_f(S,D) = \bigcup_{a \in S} \overline{E}_f(a,D)$.

Similarly, we define that f and g share the meromorphic function $\varphi(z)$ in D if $\overline{E}_f(\varphi,D) = \overline{E}_g(\varphi,D)$, where $\overline{E}_f(\varphi,D) = \{z \in D: f(z) - \varphi(z) = 0\}$, where each multiple zeros is only counted one time (ignoring multiplicity). We say that f and g share the set S_1 of distinct meromorphic functions in D if $\overline{E}_f(S_1,D) = \overline{E}_g(S_1,D)$, where $\overline{E}_f(S_1,D) = \bigcup_{\varphi \in S_1} \overline{E}_f(\varphi,D)$.

By using a geometric method and the theory of a covering surface, Sun Daochun[13] proved the following result, which extends Montel's Criterion from another direction.

Theorem 8 Let F be a family of functions meromorphic on a domain D. If for each pair of functions f, $g \in F$, f and g share three distinct values a_1, a_2, a_3 in D, then F is normal in D.

Indeed, by applying a Möbius transformation, a_1, a_2, a_3 can be replaced by $0, 1, \infty$ in Theorem 8. Xu Yan[14] proved the next general result.

Theorem 9 Let F be a family of functions meromorphic on a domain D, let $\psi(z)$ be a meromorphic function such that $\psi(z) \not\equiv 0, \infty$ in D. Suppose that

(i) for each pair function $f, g \in F$, f and g share $0, \infty, \psi(z)$ in D;

(ii) the multiplicity of $f \in F$ is larger than that of $\psi(z)$ at the common zeros or poles of f and $\psi(z)$ in D,

then F is normal in D.

Remark 6 The condition that the multiplicity of $f \in F$ is larger than that of $\psi(z)$ at the common zeros or poles of f and $\psi(z)$ in D in Theorem 9 cannot be omitted, as is shown by the following examples.

Example 3 Let $D = \{z: |z| < 1\}$, $\psi(z) = z^k$, where $k \geqslant 2$ is a positive integer, and

$$F = \{f_n(z) = nz^2, n = 2,3,\cdots, z \in D\}.$$

Clearly, for each pair functions $f_n, f_m \in F$, $f_n(z)$ and $f_m(z)$ share $0, \infty$ in D. Since $f_n(z) - \psi(z) = (n - z^{k-2})z^2$ and $f_m(z) - \psi(z) = (m - z^{k-1})z$, $f_n(z)$ and

$f_m(z)$ share $\psi(z)$ in D. But F is not normal in D.

Example 4 Let $D=\{z: |z|<1\}$, $\psi(z)=1/z^k$, where $k\geqslant 1$ is a positive integer, and

$$F=\{f_n(z)=1/nz, n=2,3,\cdots, z\in D\}.$$

Clearly, for each pair functions $f_n, f_m\in F$, $f_n(z)$ and $f_m(z)$ share $0,\infty$ in D. Since

$$f_n(z)-\psi(z)=(z^{k-1}-n)/(nz^k),$$

$$f_m(z)-\psi(z)=(z^{k-1}-m)/(mz^k),$$

$f_n(z)-\psi(z)$ and $f_m(z)-\psi(z)$ have no zeros in D, so that $f_n(z)$ and $f_m(z)$ share $\psi(z)$ in D. But F is not normal in D.

If f and g share a_1, a_2, a_3 in D, we see that f and g share the set $S=\{a_1, a_2, a_3\}$ in D. However, the converse is not true. Fang-Hong[15] (see [16]) proved the following result, which improves Theorem 8.

Theorem 10 Let F be a family of functions meromorphic on a domain $D\subset\mathbf{C}$, a_1, a_2, a_3 be distinct complex numbers. If for each pair of functions $f, g\in F$, f and g share the set $S=\{a_1, a_2, a_3\}$ in D, then F is normal in D.

Let l be a positive integer, as before, we set

$\overline{E}_f(a, D)_{l)}=\{z\in D: f(z)-a=0$, with zero multiplicity$\leqslant l\}$,

where each multiple zeros is only counted one time (ignoring multiplicity), and

$$\overline{E}_f(S, D)_{l)}=\bigcup_{a\in S}\overline{E}_f(a, D)_{l)}.$$

If $\overline{E}_f(S, D)_{l)}=\overline{E}_g(S, D)_{l)}$, we say that f and g share the set S with multiplicity$\leqslant l$ in D.

Zhang Qingcai[17] extended Theorem 10 by shaing the set S partly, as follow.

Theorem 11 Let F be a family of functions meromorphic on a domain $D\subset\mathbf{C}$, l, q be two positive integer with $q>2+2/l$. If for each pair of functions $f, g\in F$, f and g share the set $S=\{a_1, a_2, \cdots, a_q\}$ with multiplicity$\leqslant l$ in D, then F is normal in D.

Qiu Fang-Fang[18] obtained

Theorem 12 Theorem 10 is still valid if $S=\{a_1(z), a_2(z), a_3(z)\}$, where $a_1(z)$, $a_2(z)$ and $a_3(z)$ are three meromorphic functions avoiding each other in D.

Charak-Singh[19] improved the above result.

Theorem 13 Let F be a family of functions meromorphic on a domain D, $a_1(z)$, $a_2(z)$ and $a_3(z)$ be three distinct meromorphic functions in D. If

(i) for each pair of functions $f, g\in F$, f and g share the set $S=\{a_1(z), a_2(z), a_3(z)\}$ in D;

(ii) for each $f\in F$, $f(z_0)\neq a_i(z_0)$ whenever $a_i(z_0)=a_j(z_0)$ for $1\leqslant i<j\leqslant 3$ and $z_0\in D$,

then F is normal in D.

Remark 7 The following example shows that the condition (ii) is necessary in Theorem 13.

Example 5 Let $D=\{z: |z|<1\}$, $a_i(z)=z/i$ $(i=1,2,3)$, and

$$F=\{f_n(z)=2nz, n=1,2,\cdots, z\in D\}.$$

Clearly, for each pair functions $f_n, f_m\in F$, $f_n(z)$ and $f_m(z)$ share the set $S=\{z, z/2, z/3\}$ in D. But F is not normal in D. Note that $f_n(0)=a_1(0)=a_2(0)=a_3(0)$.

The next result is due to Pang Xuecheng[20].

Theorem 14 Let F be a family of functions meromorphic on D, and let a_1, a_2 and a_3 be three distinct finite complex numbers. If, for each $f\in F$, $\overline{E}_f(a_i, D)\subset\overline{E}_{f'}(a_i, D)$, i. e. $f(z)=a_i\Rightarrow f'(z)=a_i$ $(i=1,2,3)$ in D, then F is normal in D.

Letting $\overline{E}_f(a_i, D)=\varnothing$, i. e. $f\neq a_i$ $(i=1,2,3)$, Montel's criterion follows immediately from Theorem 14.

For holomorphic case, Xu-Qiu[21] extended the above result, as follow

Theorem 15 Let F holomorphic in D, and let $a(z)$, $b(z)$ holomorphic in D. For each $f\in F$, if

(i) $a(z)\not\equiv b(z)$;

(ii) $a(z)\not\equiv a'(z)$ or $b(z)\not\equiv b'(z)$;

(iii) $f(z)-a(z)$ and $f(z)-b(z)$ have no common zeros;

(iv) $f(z)=a(z)\Rightarrow f'(z)=a(z)$, and $f(z)=b(z)\Rightarrow f'(z)=b(z)$,

then F is normal in D.

Remark 8 The hypothesis $a(z)\not\equiv b(z)$ cannot be omitted in Theorem 15, as is shown by following example.

Example 6 Let $D=\{z: |z|<1\}$ and $a(z)=b(z)=1$, and let

$$F=\{f_n(z)=e^{nz}+1: n=1,2,\cdots,z\in D\},$$

Clearly, for each $f_n\in F$, $f_n(z)\neq 1$. Hence, all conditions except for condition (i) in Theorem 1 are satisfied. However, F is not normal in D.

Remark 9 The next example shows that the condition (ii) is necessary in Theorem 15.

Example 7 Let $D=\{z: |z|<1\}$ and $a(z)=-e^z$, $b(z)=e^z$, and let

$$F=\{f_n(z)=e^z\sin(nz): n=1,2,\cdots,z\in D\},$$

Clearly, for each $f_n\in F$, we have $f_n(z)=e^z\Rightarrow f'_n(z)=e^z$ and $f_n(z)=-e^z\Rightarrow f'_n(z)=-e^z$. Then condition (i), (iii) and (iv) in Theorem 1 are satisfied. But F is not normal in D. In fact, $f^{\#}_n(0)=n/2\to\infty$ as $n\to\infty$, where $f^{\#}(z)=|f'(z)|/(1+|f(z)|^2)$ is the spherical derivative of f. By Marty's criterion, F is not normal in D.

Remark 10 The condition (iii) is required for Theorem 15 to hold, as is shown by following example.

Example 8 Let $D=\{z: |z|<1\}$, $k(\geqslant 3)$ be a positive integer, and $a(z)=z^k$, $b(z)=2z^k$, and let

$$F=\{f_n(z)=nz^2+z^k: n=1,2,\cdots,z\in D\},$$

Clearly, for each $f_n\in F$, $f_n(z)=a(z)\Rightarrow f'_n(z)=a(z)$, and $f_n(z)=b(z)\Rightarrow f'_n(z)=b(z)$ in D. But, it is easy to see that F is not normal in D.

For the case of families of meromorphic functions, we ask

Question 2 Does Theorem 14 still hold if distinct values a_i $(i=1,2,3)$ are replaced by distinct meromorphic functions $a_i(z)$ $(i=1,2,3)$?

As we know, Miranda's criterion is another well-known result on normal families of holomorphic functions, as follow

Miranda's Criterion Let F be a family of functions holomorphic on a domain D, k be a positive integer. If, for each $f\in F$, $f\neq 0$, $f^{(k)}\neq 1$ in D, then F is normal in D.

In [22], Li Baoqin proved the following joint theorem.

Theorem 16 Let F be a family of functions holomorphic in D, and let a, b be nonzero complex numbers, and A, B positive constants. Suppose that for all f, $g\in F$,

(i) $A<|f'(z)|<B$ and $|f''(z)|<B$ whenever $z\in f^{-1}(0)$;

(ii) $\min\{|f''(z_1)|, |g'(z_2)|\}\leqslant B$ whenever $(z_1,z_2)\in F^{-1}(0)$, where $F(z_1,z_2)=(f'(z_1)-a, g(z_2)-b)$,

then F is normal in D.

Remark 11 For each $f\in F$, if $f\neq 0$, b in Theorem 16, then $f^{-1}(0)=F^{-1}(0)=\varnothing$, and thus conditions (i) and (ii) hold vacuously. So Theorem 16 generalizes Montel's criterion. If $f\neq 0$, $f'\neq a$ for each $f\in F$ in Theorem 16, then conditions (i) and (ii) also hold vacuously. Therefore, Theorem 16 extends Miranda's criterion too.

In addition, some extensions and generalizations of Montel's criterion for families of holomorphic mappings of a domain D in $\mathbf{C}$ into $P^N(\mathbf{C})$ are obtained during past years.

2 Reference

[1] Hayman W K. Meromorphic functions [M]. Oxford: Clarendon Press, 1964.

[2] Schiff J L. Normal families [M]. New York, Berlin: Springer-Verlag, 1993.

[3] Yang Le. Value distribution theory [M]. Berlin: Springer-Verlag, Science Press, 1993.

[4] Montel P. Lecons sur les familles normales de fonctions analytiques et leurs applications [M]. Paris: Gauthier-Villars, 1927.

[5] Bargmann D, Bonk M, Hinkkanen A, et al. Families of meromorphic functions avoiding continuous functions [J]. J Anal Math, 1999, 79(1): 379-387.

[6] Chang Jianming, Fang Mingliang, Zalcman L. Composite meromorphic functions and normal families [J]. Proc Roy Soc Edinburgh: Ser A, 2009, 139(1): 57-72.

[7] Xu Yan. Another improvement of Montel's criterion [J]. Kodai Math J, 2013, 36(1): 69-76.

[8] Xu Yan. On Montel's theorem and Yang's problem [J]. J Math Anal Appl, 2005, 305(2): 743-751.

[9] Bloch A. Les fonctions holomorphes et méromorphes dans le cercle unite [M]. Paris: Gauthier-Villars, 1926.

[10] Valiron G. Familles normales et quasi-normales de fonctions méromorphes [M]. Paris: Gauthier-Villars, 1929.

[11] Carathéodory C. Theory of functions of a complex variable

[M]. New York: Chelsea Publ Co, 1960.

[12] Grahl J, Nevo S. Exceptional functions wandering on the sphere and normal families [J]. Israel J Math, 2014, 202(1): 21-34.

[13] Sun Daochun. On the normal criterion of share value [J]. Journal of Wuhan University: Natural Science Edition, 1994, 40(3): 9-12.

[14] Xu Yan. Montel's criterion and shared function [J]. Publ Math Debrecen, 2010, 77(3): 471-478.

[15] Fang Mingliang, Hong Wei. Some results on normal family of meromorphic functions [J]. Bull Malays Math Sci Soc, 2000, 23(2): 143-151.

[16] Sun Daochun, Liu Huifang. Normal criteria on the family of meromorphic functions with shared set [J]. Israel J Math, 2011, 184(1): 403-412.

[17] Zhang Qingcai. Normal criteria concering sharing values [J]. Kodai Math J, 2002, 25(1): 8-14.

[18] Qiu Huiling, Fang Caiyun, Fang Mingliang. Normal families concerning exceptional functions and shared functions [J]. Acta Math Sinica: Chinese Ser, 2015, 58(2): 345-352.

[19] Charak K S, Singh V. Sharing of a set of meromorphic functions and Montel's theorem [J]. Arch Math, 2016, 106(3): 257-263.

[20] Pang Xuecheng. Normal families and normal functions of meromorphic functions [J]. Chinese Ann Math: Ser A, 2000, 21(5): 601-604.

[21] Xu Yan, Qiu Huiling. An improvement of Montel's criteria [J]. J Math Anal Appl, 2008, 343(2): 1075-1079.

[22] Li Baoqin. A joint theorem generalizing the criteria of Montel and Miranda for normal families [J]. Proc Amer Math Soc, 2004, 132(9): 2639-2646.

Montel 正规定则的推广与改进

徐 焱

(南京师范大学数学科学学院,江苏 南京 210023)

摘要: Montel 正规定则是亚纯函数正规族理论中最著名的结果,它在亚纯函数值分布、复动力系统、极值问题等方面有着重要的应用. 至今,已得到它的许多推广与改进. 该文将介绍 Montel 正规定则的一些推广与改进,同时给出了关于 Montel 正规定则的 2 个问题.

关键词: Montel 正规定则; 正规族; 例外值; 分担值

(责任编辑: 王金莲)